Basics of mechanisms

Kinematics

study of relative motion between the various parts of the machines

Dynamics

- deals with the forces and their effects, while acting upon the machine parts in motion

Kinetics

- deals with the inertia forces which arise from the combined effects of the mass and motion of the machine parts.

Statics

- deals with the forces and their effects while the machine parts are at rest. The mass of the parts is assumed to be negligible.

Link

Each part of a machine which moves relative to some other parts is known as link.

properties:- It should have relative motion
It must be a resistant body

Types of Links

Rigid Link:- which does not undergo any deformation while transmitting motion.

Eg: Reciprocating steam engine CR, crank

Flexible Link:- which is partly deformed in a manner not to affect the transmission of motion

Eg:- belts, rops, chains & wires, transmit tensile force only.

Fluid link:- which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only in the case of hydraulic presses, jacks and brakes.

Kinematic pair

The two links or elements of a machine when in contact which each other are said to form a pair.

If the relative motion between them is completely constrained (definite direction), the pair is known as kinematic pair.

Types of constrained motions

1. completely constrained motion:

when the motion btn the pair is limitted a definite direction irrespective of the direction of force applied.

Eg:- piston & cylinder, square bar in a square hole
shaft with collars in a circular hole

2. Incompletely constrained motion

when the motion btn the pair can take place in more than one direction.

Eg: shaft in a circular hole → either rotate or slide in a hole

3. successfully constrained motion

when the motion btn the element forming a pair, is such that the constrained motion is not completed by itself, but by some other means.

Eg:- shaft in a foot-step bearing, the motion of a
I.C. Engine valve.

Classifications of kinematic pairs

1. according to the types of relative of motion btn the elements

sliding pair:- Eg:- ram and its guides in a shaper, Tail stock on the lathe bed.

...ing pair — cycle wheels turning over their axles, ... supported in head stock.

Rolling Pair – Ball and roller bearing

Screw Pair – bolt with nuts, the lead screw of a lathe with nut

Spherical Pair – ball and socket joint, pen stand, attachment of a car mirror.

2. According to the type of contact btn the element

Lower Pair – surface contact → Eg: sliding, Rolling and screw pair

Higher Pair – Line or Point contact → Eg: belt and rope drives, ball and roller bearing.

3. According to the type of closure

self closed pair – lower pairs

Force closed pair – Cam and follower

Kinematic chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, it is called a kinematic chain.

$$l = 2P - 4$$

$$j = \frac{3}{2}l - 2$$

l – Num of links

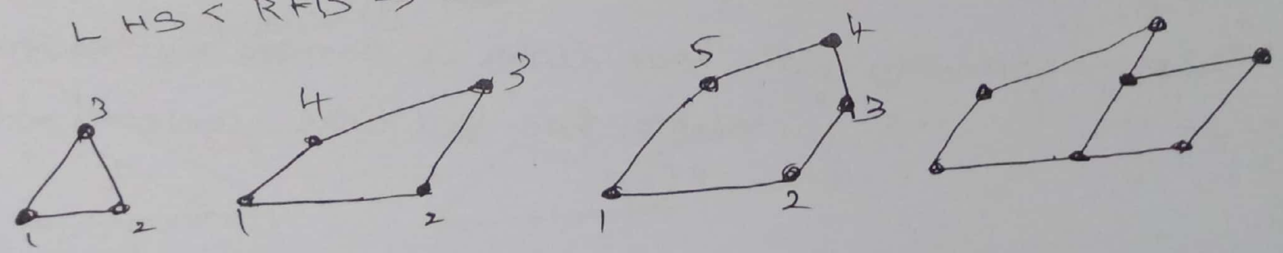
P – Number of pairs

j – number of joints

L.H.S = RHS → Kinematic chain or constrained kinematic chain

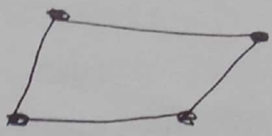
L.H.S > RHS → Locked chain

L.H.S < RHS → unconstrained kinematic chain



Types of joints in chain

1. Binary joint → when the two links are joined at the same connection



A.W. klein

$$j + \frac{h}{2} = \frac{3}{2}l - 2$$

j - number of binary joints

h - Number of higher pairs

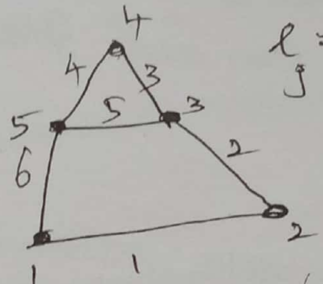
l - number of links

Types of kinematic chain

1. Four bar or quadric cycle chain
2. single slider crank chain
3. Double slider crank chain

2. Ternary joint → when the three links are joined at the same connection

1 Ter = 2 Binary

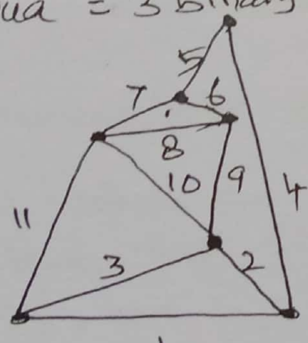


$l = 6$

$j = 7$

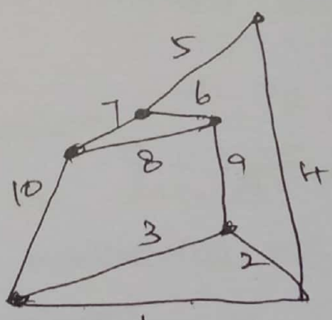
4. Quatornary joints → when the four links are joined at the same connection

1 Qua = 3 binary



$l = 11$

$j = 15$



Mechanism

when one of the links of kinematic chain is fixed the chain is known as mechanism.

Eg:- Typewritter, engine indicator.

- A mechanism with four links → simple mechanism
- " " more than four links - compound mechanism

Machine

When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine.

Structure

It is an assemblage of number of resistant bodies having no relative motion btn them and meant for carrying load having straining action.

Eg:- railway bridge, roof truss, machine frames.

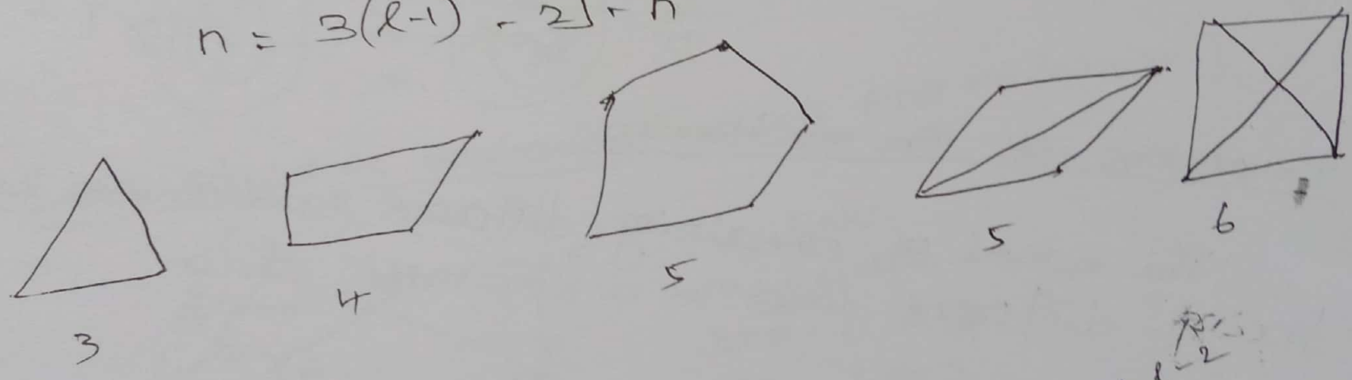
DOF (or) movability

Number of input parameters which must be independently controlled in order to bring the mechanism into useful engineering purpose.

The number of degrees of freedom $n = 3(l-1) - 2j - h$

Kutzbach criterion

$$n = 3(l-1) - 2j - h$$



$j = \frac{3}{2}l - 2$

$\frac{3}{2} \times 3 - 2$

$\frac{9}{2} - 2$

$4.5 - 2$

2.5

$n = 1$ one input

$n = 2$ Two separate input

$n = 0$ structure

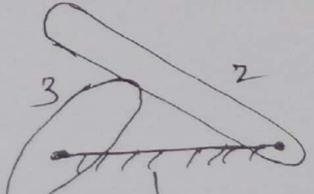
$n = -1$ statically indeterminate structure

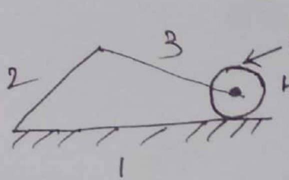
$j + \frac{h}{2} = \frac{3}{2}l - 2$

$j + \frac{1}{2} = \frac{3}{2} \times 4 - 2$

$j = 4 - 0.5$

$= 3.5$





$l = 4$
$j = 3$
$h = 1$

$l = 3$
$j = 2$
$h = 1$

Grubler's criterion

put $n=1$ in kutzbach equation and $h=0$

$$n = 3(l-1) - 2j - h$$

$$1 = 3(l-1) - 2j$$

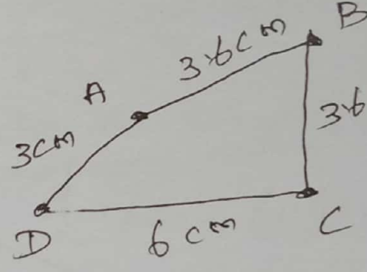
$$1 = 3l - 3 - 2j$$

$$3l - 2j - 4 = 0$$

Eg: slider crank mechanism

Grashoff's law

The sum of the shortest and longest link lengths should not be greater than the sum of remaining two link lengths.



$3+6 = 3.6+3.$

$9 = 7.2$

$9 > 7.2$

$150 > 125$

Inversions of the mechanism

The method of obtaining different mechanism by fixing different links in a kinematic chain.

Mechanical Advantage

The ratio of output torque to input torque or the ratio of load to effort

$$M.A = \frac{T_B}{T_A} = \frac{\omega_A}{\omega_B}$$

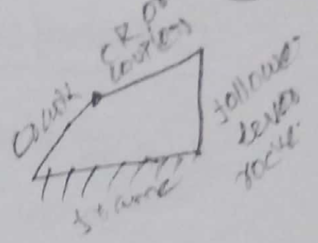
$$MA = \frac{T_B}{T_A}$$

$$M.A_{actual} = \eta \times \frac{T_B}{T_A}$$

(4)

Transmission angle (γ)

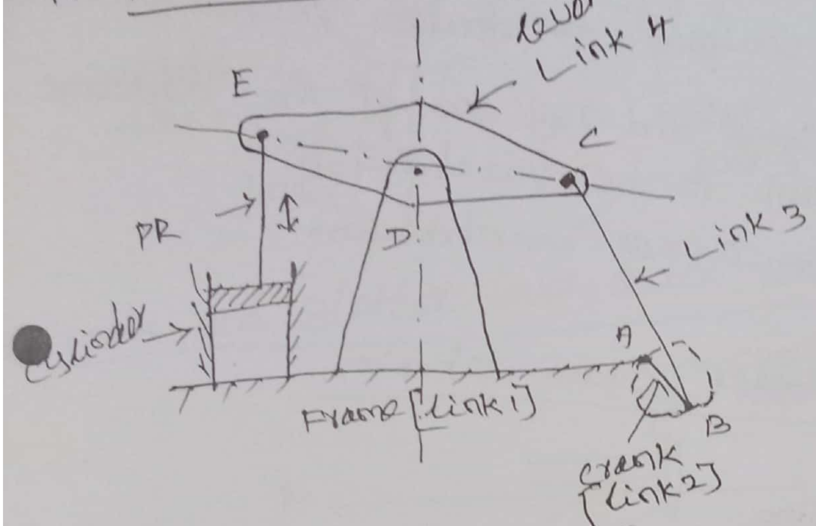
The angle btn coupler and follower.



Inversion of Four bar chain

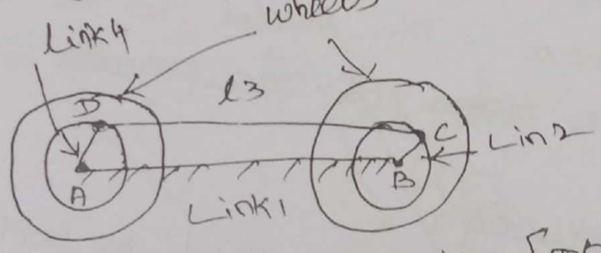
1. Beam Engine (crank and lever mechanism)

Rotary motion → Reciprocating motion

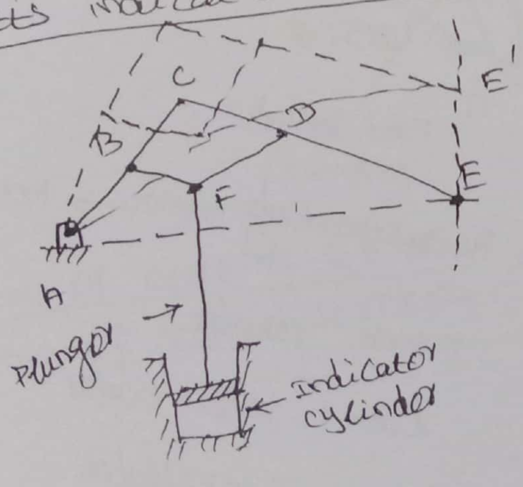


2. Coupling rod of a locomotive [Double crank mechanism]

Rotary motion one wheel to another wheel.



3. watt's indication mechanism [Double lever mechanism]

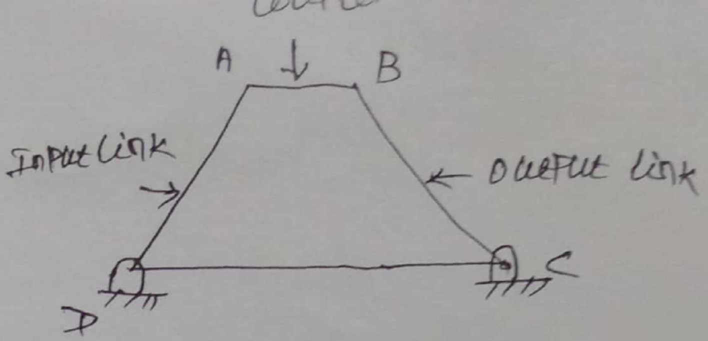


A - link1
AC - link 2
BFD - link 4
CE - link 3

CE, BFD - Act as levers

Grashof's law

$l+s \leq p+q$



Inversions

1). when shortest link is fixed - double crank mechanism

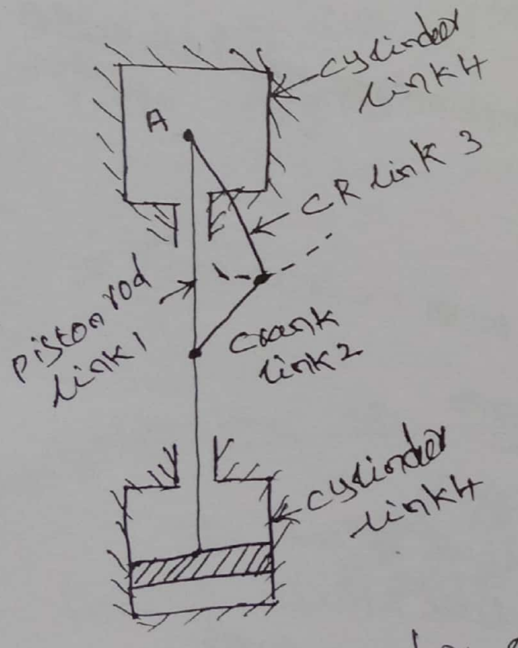
2). when shortest link is the coupler → double rocker mechani

3). when sortest link is the crank and any one of the adjacent links is fixed → crank rocker mechanism

4). $l+s > p+q$ → rocker - rocker mechanism

Inversions of single slider crank chain

pendulum pump or Bull engine

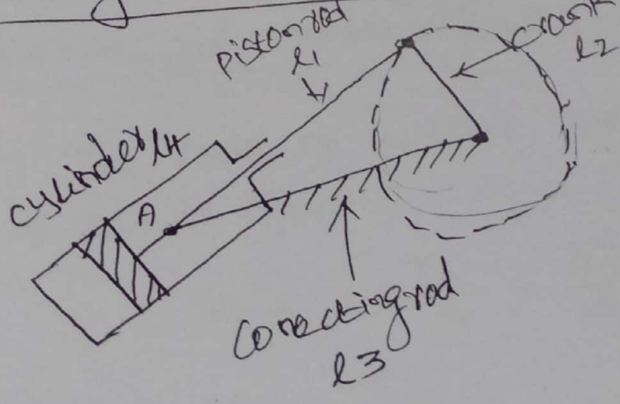


link 2 rotates → connecting rod oscillates about a pin pivoted to the link 4 at A. The piston rod reciprocates.

Application

Duplex pump → used to supply feed water to boilers.

oscillating cylinder engine



Reciprocating motion → Rotary motio

crank rotates

link 1 reciprocates

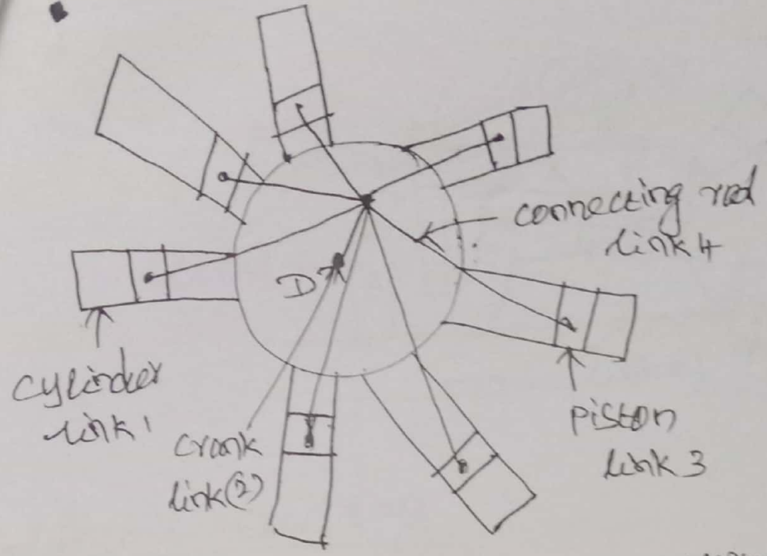
link 4 oscillates

ary internal combustion Engine (or) Gnome engine

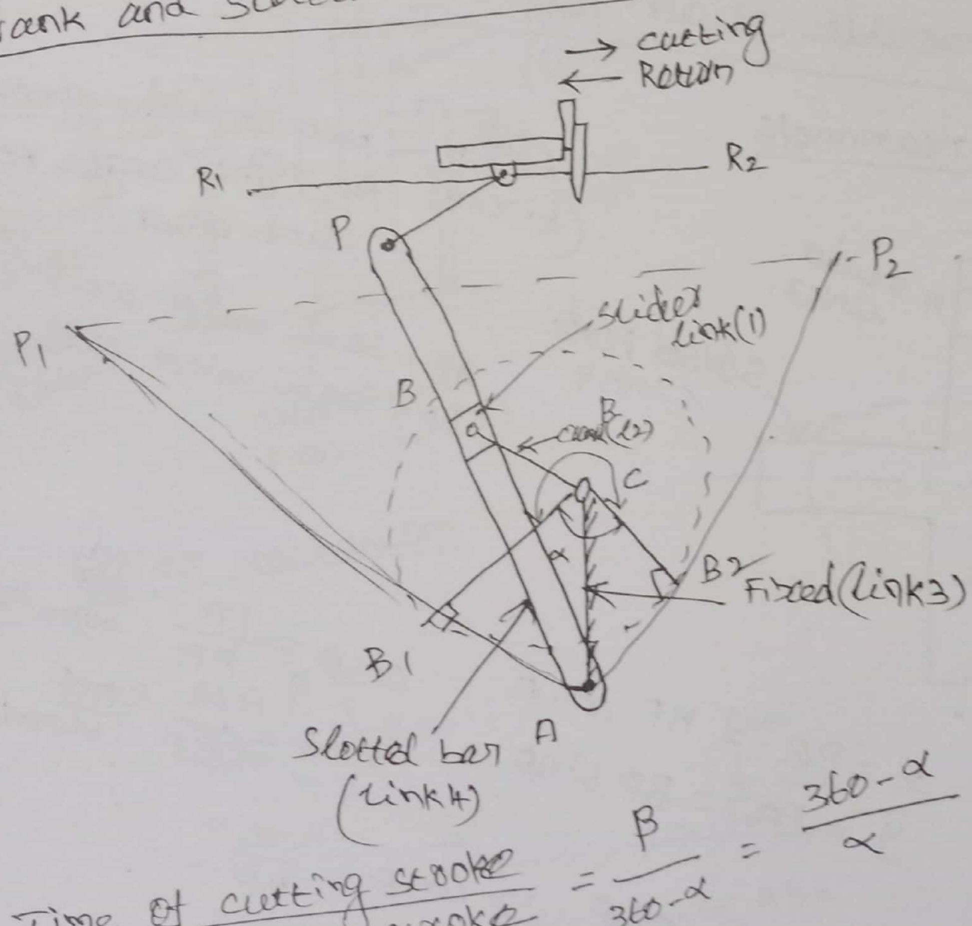
Crank fixed

Connecting rod rotates

Piston reciprocates

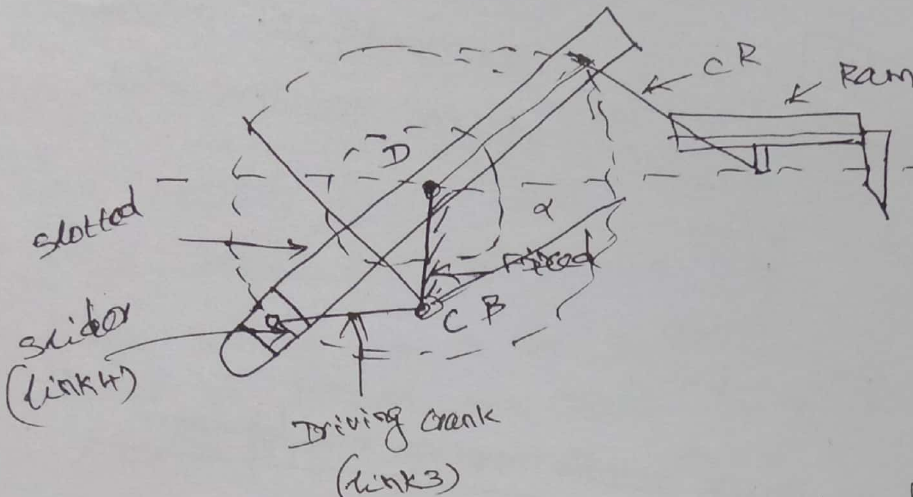


Crank and slotter lever Quick return motion mechanism



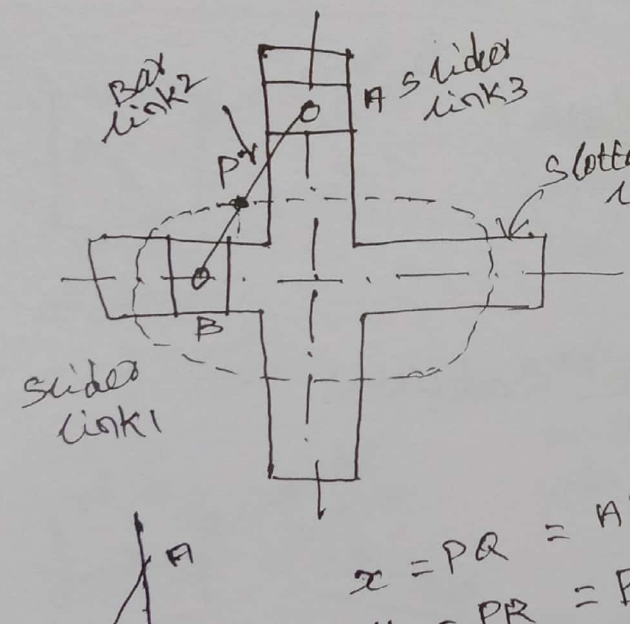
$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{360-\alpha} = \frac{360-\alpha}{\alpha}$$

Whitworth quick return motion mechanism



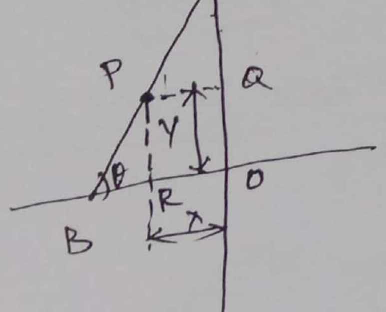
Inversion of Double slider crank chain Two turning two sliding pair

1). Elliptical trammels



link4 (fixed) - two straight grooves at right angles to each other

AP - semi major axis
BP - semi minor axis



$x = PQ = AP\cos\theta$ $\qquad \cos\theta = \frac{PQ}{AP}$ $\quad \frac{\text{adj}}{\text{Hyppon}}$

$y = PR = BP\sin\theta$ $\qquad \sin\theta = \frac{PR}{BP}$ $\quad \frac{\text{opp}}{\text{Hypoten}}$

$x = AP\cos\theta \Rightarrow \cos\theta = \frac{x}{AP}$

$y = BP\sin\theta \Rightarrow \sin\theta = \frac{y}{BP}$

squaring and adding

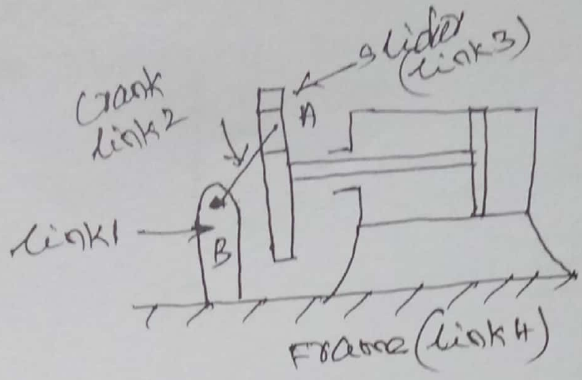
$$\frac{x^2}{AP^2} + \frac{y^2}{BP^2} = \cos^2\theta + \sin^2\theta$$

$$\boxed{\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = 1}$$

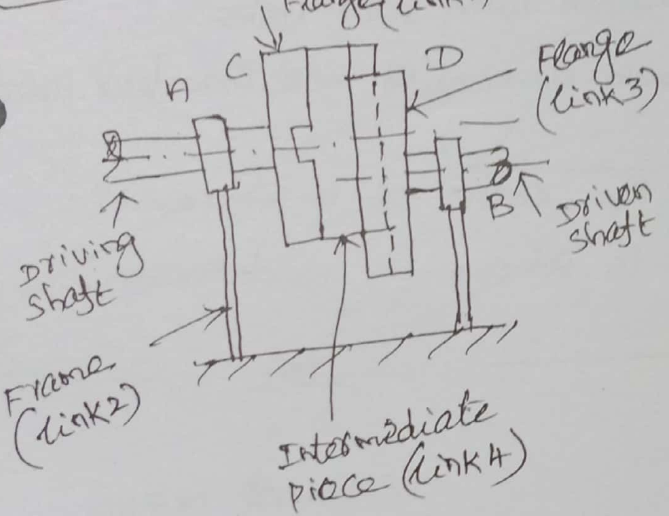
(1)

Scotch Yoke mechanism

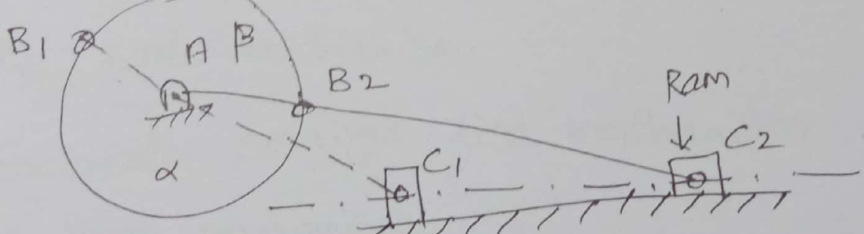
Rotary → Reciprocating



3). oldham's coupling



offset slider mechanism



Forward stroke → Angle AB_1 to AB_2

Return stroke → Angle AB_2 to AB_1

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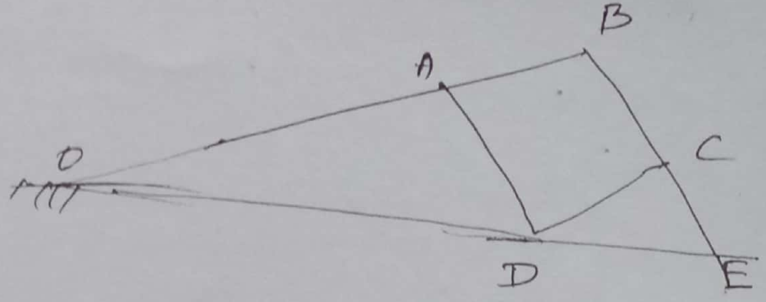
Pantograph



Classifications of mechanism

1) simple mechanism ⇒ mechanism with four links
2). compound mechanism ⇒ mechanism with more than four links

mechanism:- when one of the links of a kinematic chain is fixed, the chain is known as mechanism.

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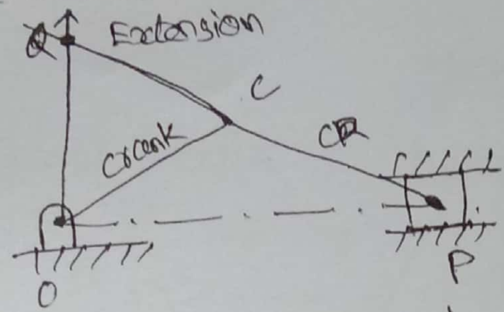
Straight line generators

1). Copied straight line motion mechanism

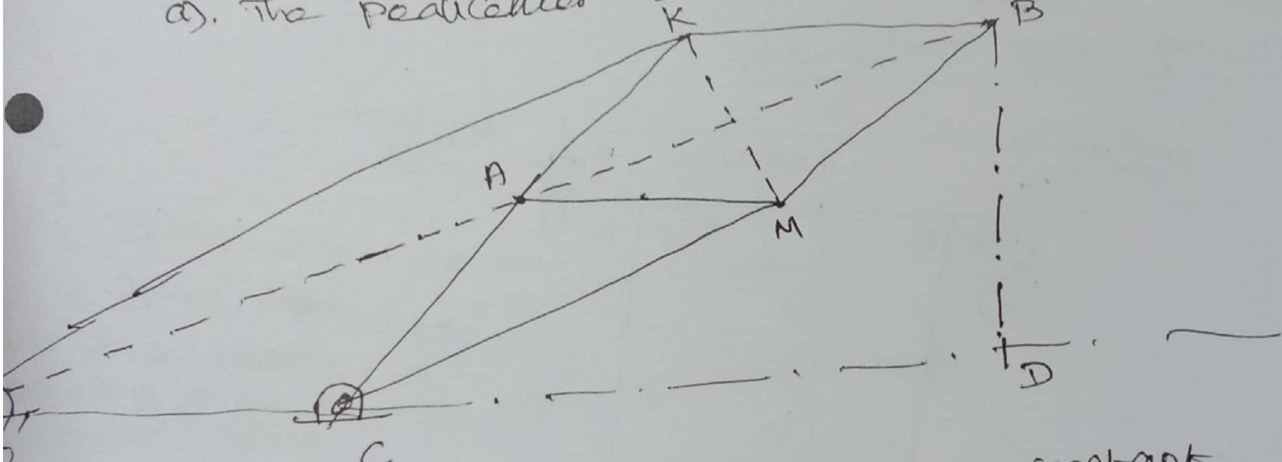
a). The scott Russol mechanism



PC = CQ = OC

2). Exact straight Line mechanism

a). The peaucellier mechanism



OA×OB = constant

AKBM rhombus

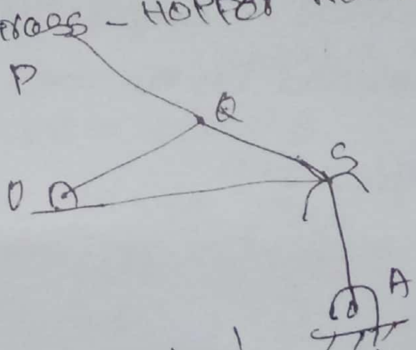
B traces a straight path

b). The Hart's mechanism

3). Approximate straight line mechanism

a). The watt mechanism

b). Grass - Hopper mechanism



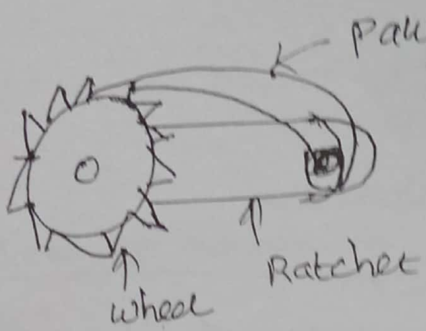
c). Tchebicheff's mechanism

Toggle or snap action of ...

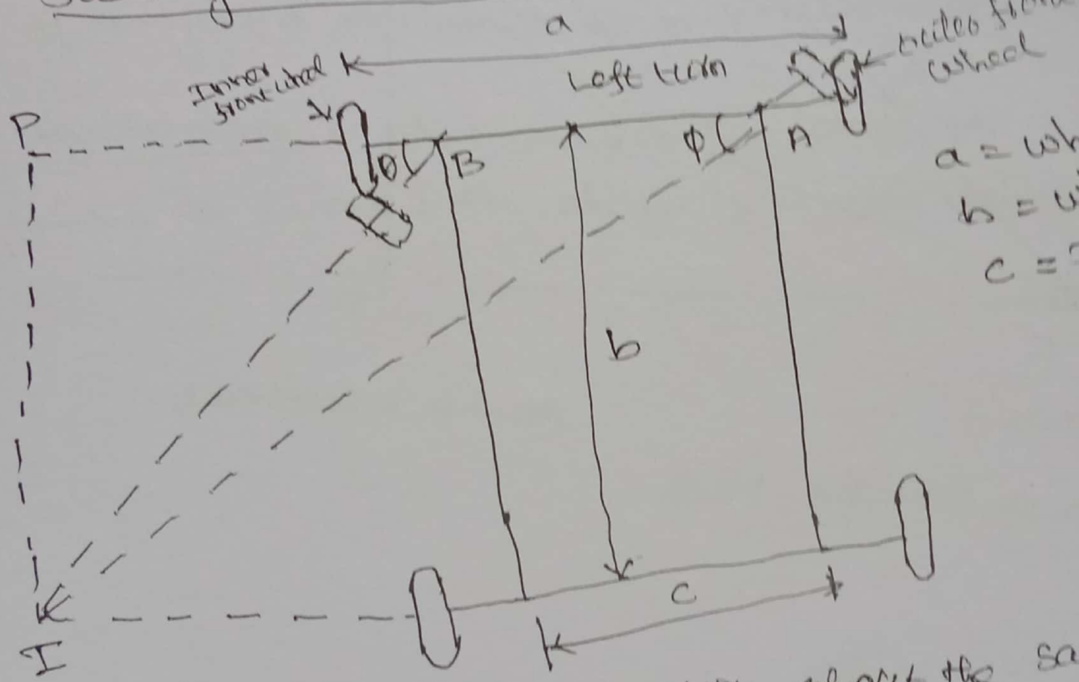
Bicycle bell, switches, circuit breakers, clamps, spring clips.

Ratchets and escapements

lifting jacks



Steering gear mechanism



a = wheel track
b = wheel base
c = Distance b/w the pivot A and B of the front axle.

* The two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels.
* Condition for correct steering → all the four wheels must turn about the same instantaneous centre.
* The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.

Triangle IBP

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{IP}{BP}} = \frac{BP}{IP}$$

Triangle IAP

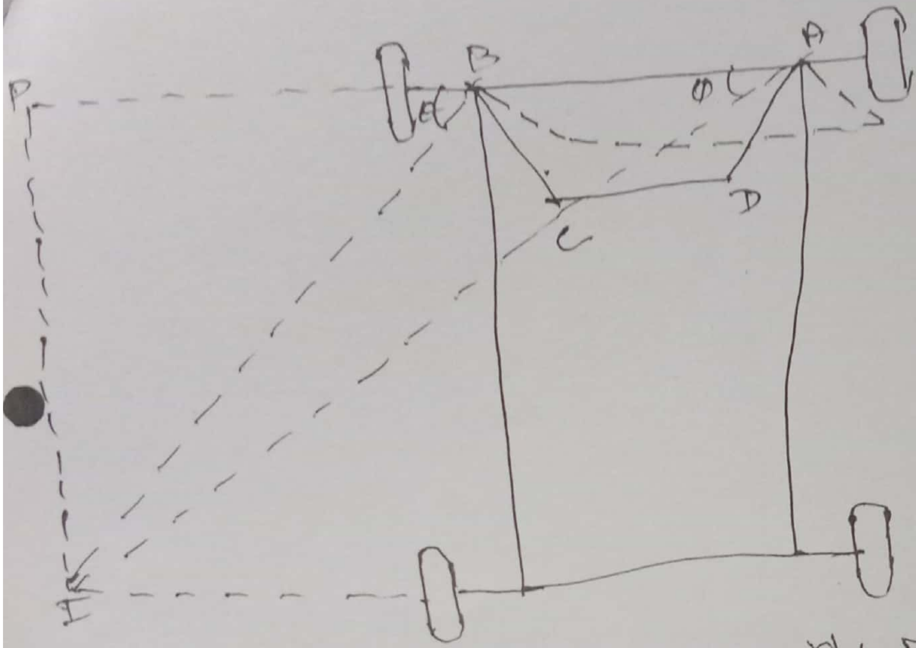
$$\cot\phi = \frac{AP}{IP} = \frac{AB+BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP}$$

$$\cot\phi = \frac{c}{b} + \cot\theta$$

$$\boxed{\cot\phi - \cot\theta = c/b}$$

1). Davis steering Gear
2). Ackerman steering Gear

Ackerman Steering gear



Consists of turning pair
ABCD – Four bar chain
BC & AD → shorter links equal length
AB & CD → long unequal length.

1). Vehicle moves along a straight path
The links AB & CD are parallel
BC & AD are equally inclined to the longitudinal axis of the vehicle.

2). For left turn and right turn.
All the four wheel must turn about the same instantaneous centre.

Akerman	Davis
1). The whole mechanism is on the back of the front wheels	It is in front of the wheels
2). Consist of turning pairs	consists of sliding members.

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CAMS

Cam:

: cam is a mechanical member used to impart desired motion to a follower by direct contact. (Higher pair)

The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating.

Necessary elements of a cam mechanism are

- A driver member known as the cam
- A driven member called the follower
- A frame which supports the cam and guides the follower

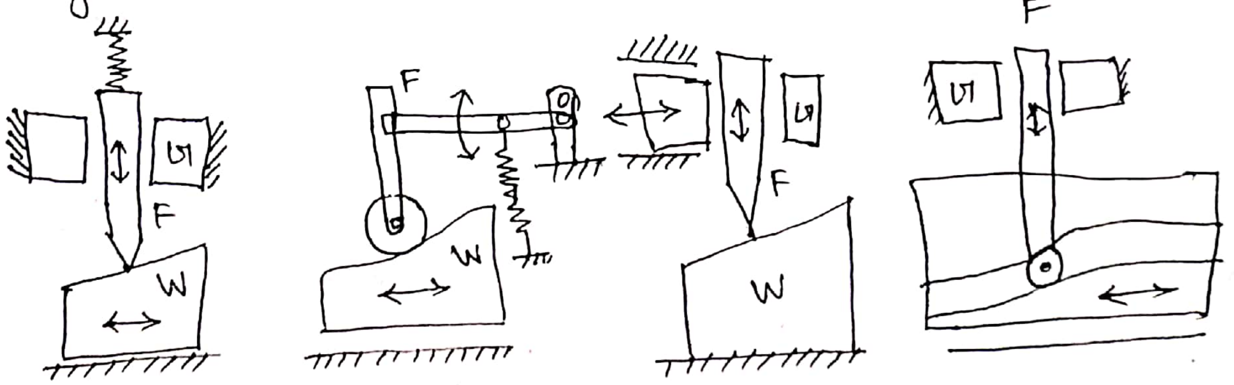
Types of cam

according to

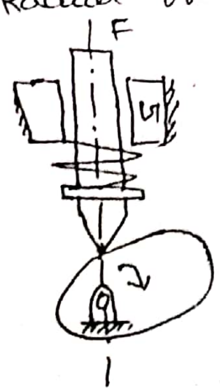
1. shape
2. Follower movement
3. Manner of constraint of the follower

According to shape

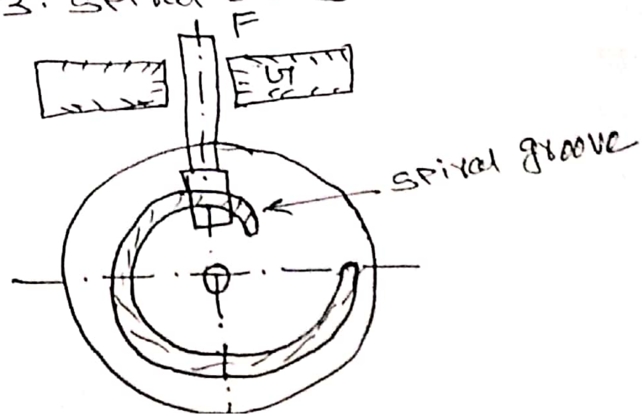
1. wedge and Flat cams



2. Radial or disc Cams

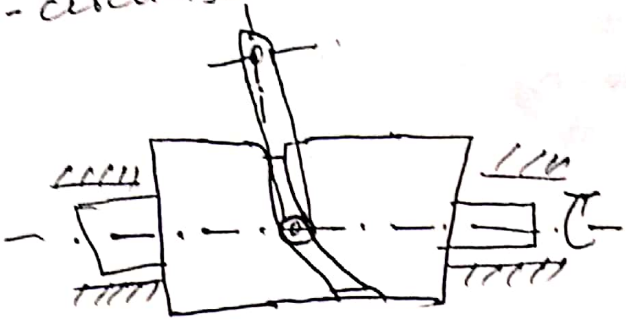


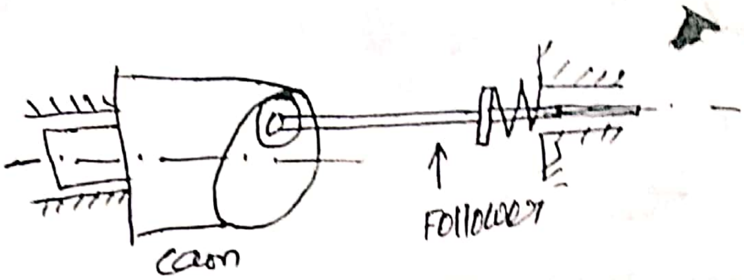
3. spiral cams



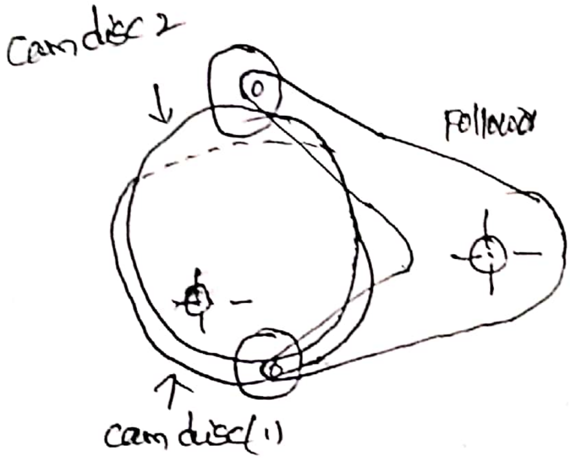
4. cylindrical cams [barrel or drum cams]

- circumferential contour cut in the surface

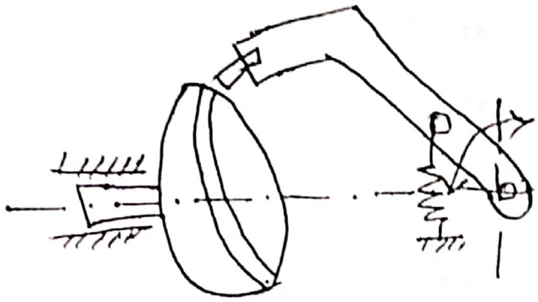




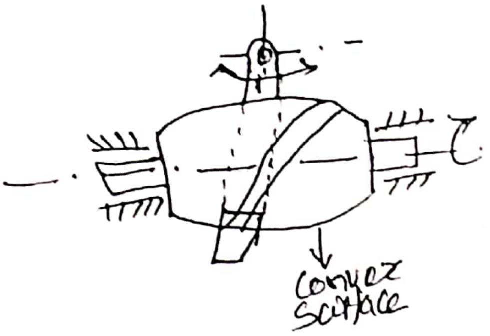
5. conjugate cam

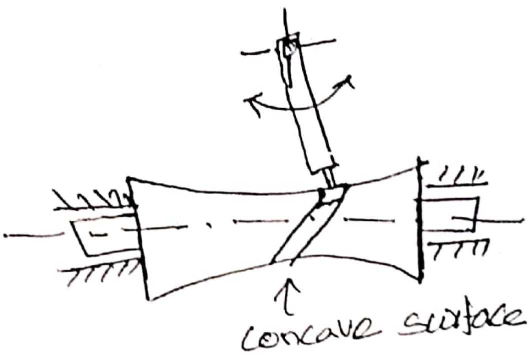


6. spherical cams



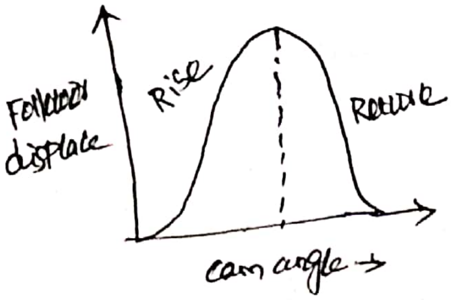
7. globoidal cams



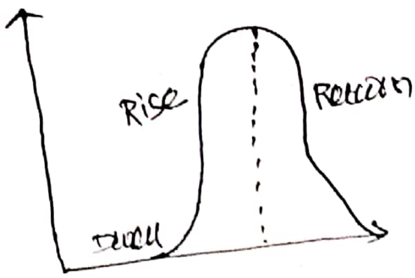


According to Follower movement

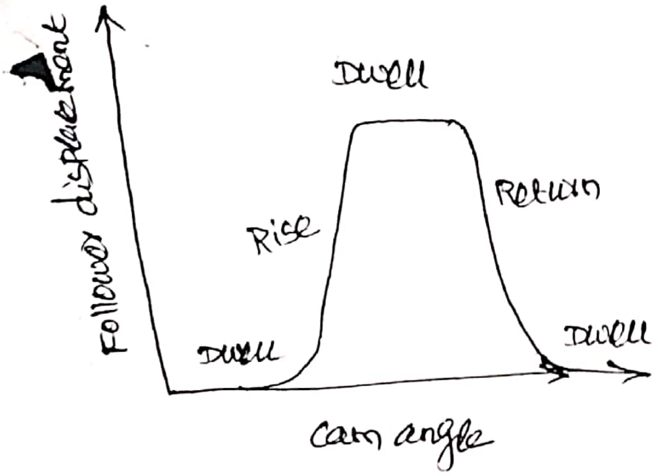
1. Rise - Return - Rise



2. Dwell - Rise - Return - Dwell



4 — Rise – Dwell – Return – Dwell



According to manner of constraint of the follower

1. pre-loaded spring cam
2. positive drive cam
3. Gravity cam

Types of followers

Followers are classified according to the

1. shape
2. movement
3. location of line of movement

According to shape

1. knife-edge follower 2. Roller follower 3. Mushroom follower

According to movement

1. Reciprocating follower 2. oscillating follower

According to location of line of movement

1. Radial follower
2. offset follower

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Angle of Ascent (ϕ_a)

It is the angle through which the cam turns during the time the follower rises.

Angle of Dwell (δ)

The angle of the dwell is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.

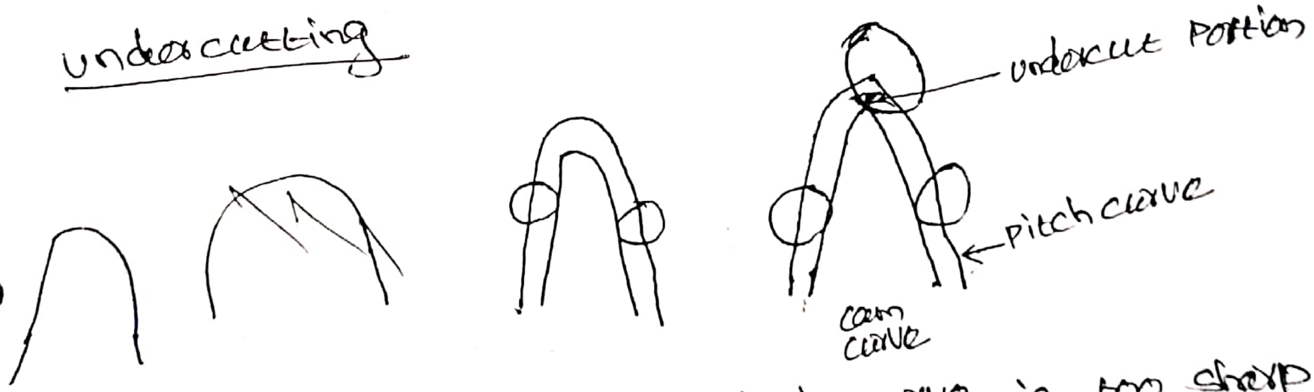
Angle of Descent (ϕ_d)

It is the angle through which the cam turns during the time the follower returns to the initial position.

Angle of Action

The angle of action is the total angle moved by the cam during the time, btn the begining of rise and the end of the return of the follower.

Undercutting



If the curvature of the pitch curve is too sharp, then the part of the cam shape would be lost. This is called undercutting.

Motions of the follower

1. Uniform velocity
2. Simple harmonic motion
3. Uniform acceleration and retardation
4. Cycloidal motion.

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Simple Harmonic motion

1. Shanmuga Priyan
2. Tamilarasan
3. Karthick anand
4. Gowtham
5. Karthick
6. madhan
7. Logesh Kumar
8. S.R. Rajkumar
9. Ashok
10. Gnana Barathi
11. Shanmugananda Ramkumar
12. Yoganandham

13, ~~33~~, 63

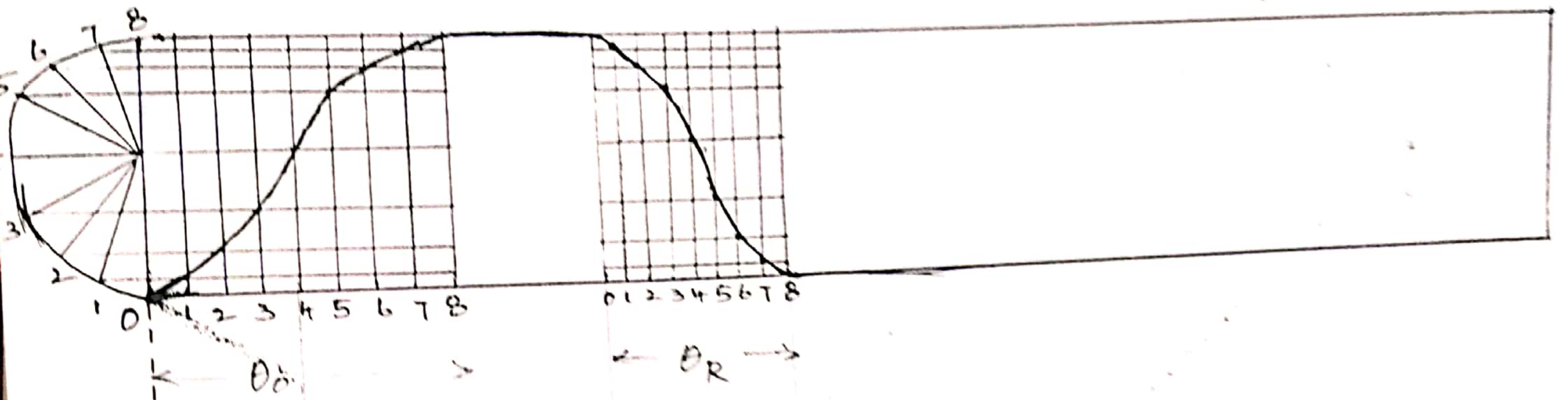
2H - for hard works
H - drawings & lettering
2H - construction lines, dimension lines, section lines, ce

HB - medium soft
H, 2H - degree of hardness
B → 2B, 3B → degree of softness

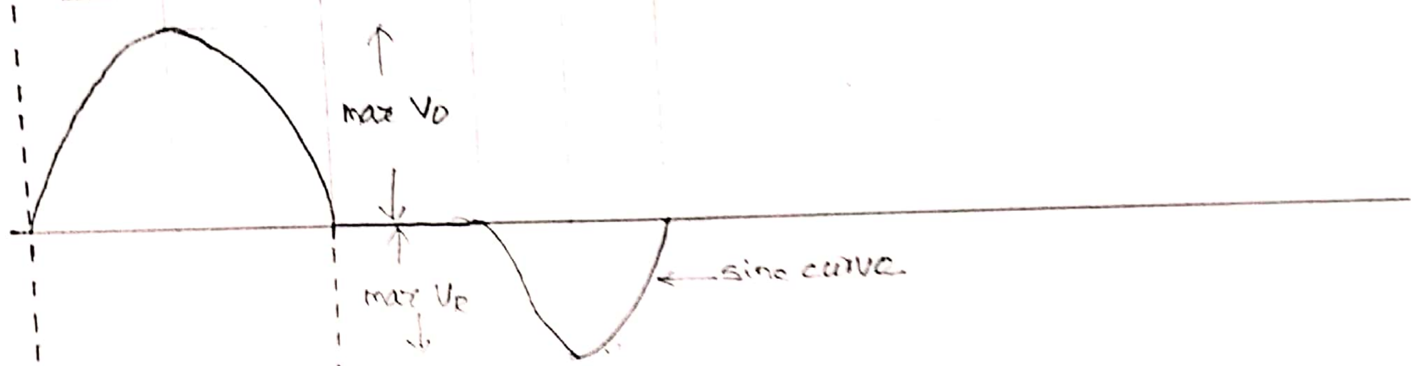
Scanned by CamScanner

Simple harmonic motion

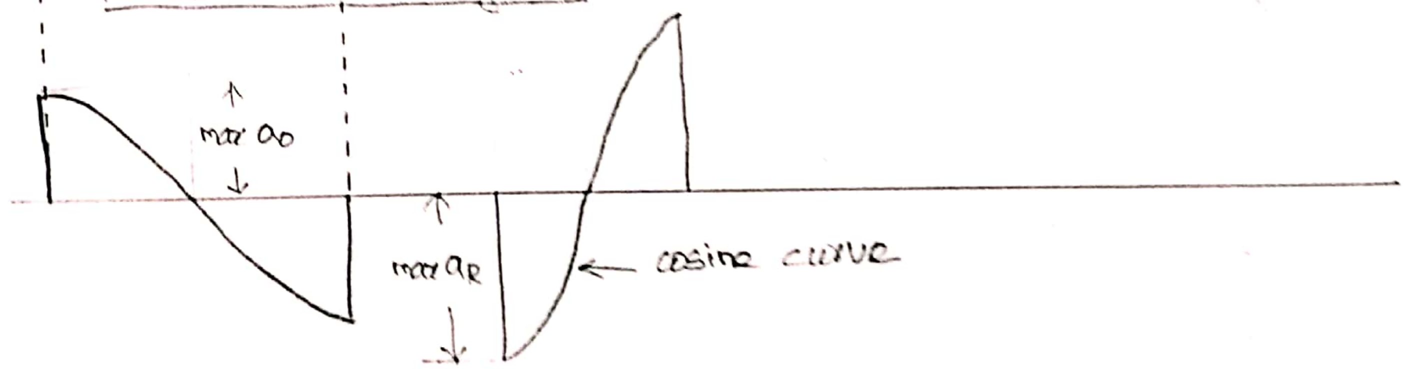
* Displacement diagram



Velocity diagram



Acceleration diagram



During out stroke

$$V_o = \frac{\pi \omega S}{2\theta_o} \qquad a_o = \frac{\pi^2 \omega^2 S}{2(\theta_o)^2}$$

During return stroke

$$V_R = \frac{\pi \omega S}{2\theta_R} \qquad a_R = \frac{\pi^2 \omega^2 S}{2(\theta_R)^2}$$

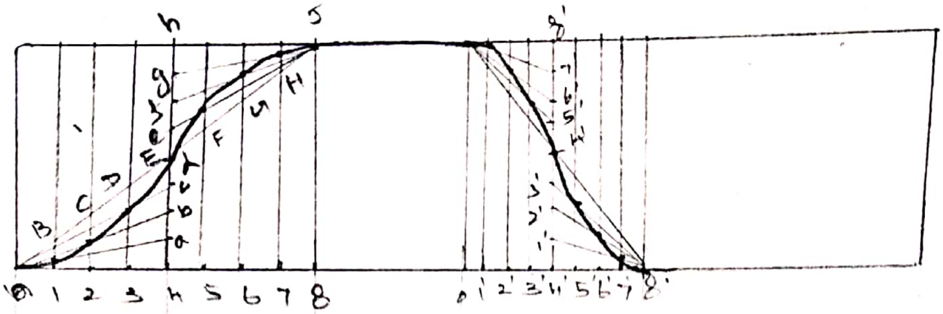
S – stroke of the follower

θ_o & θ_R – angular displacement of the cam during out stroke and return stroke of the follower in radians

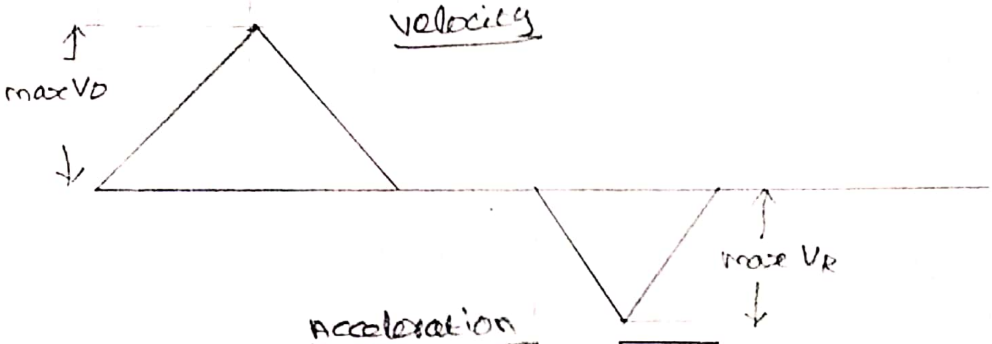
ω - Angular velocity of the cam in rad/sec

Uniform Acceleration and Retardation

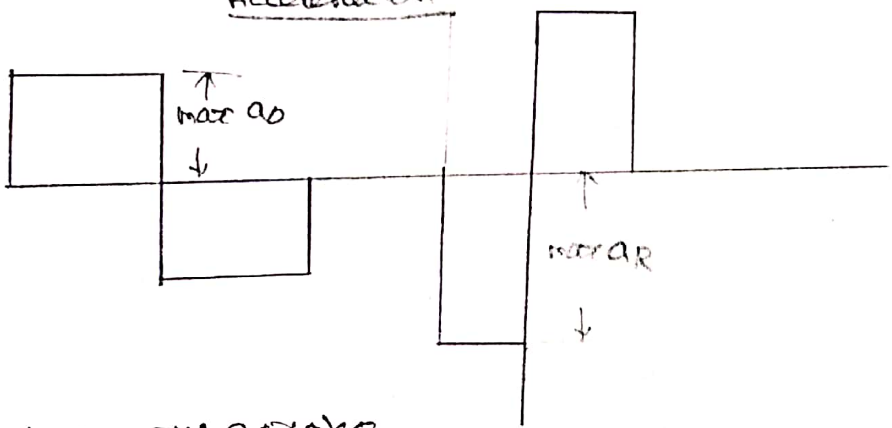
scale 1° = 2mm



<u>velocity</u>



<u>Acceleration</u>



<u>During outstroke</u>

$$V_o = \frac{2\omega S}{\theta_o} \quad , \quad a_o = \frac{4\omega^2 S}{(\theta_o)^2}$$

<u>During returnstroke</u>

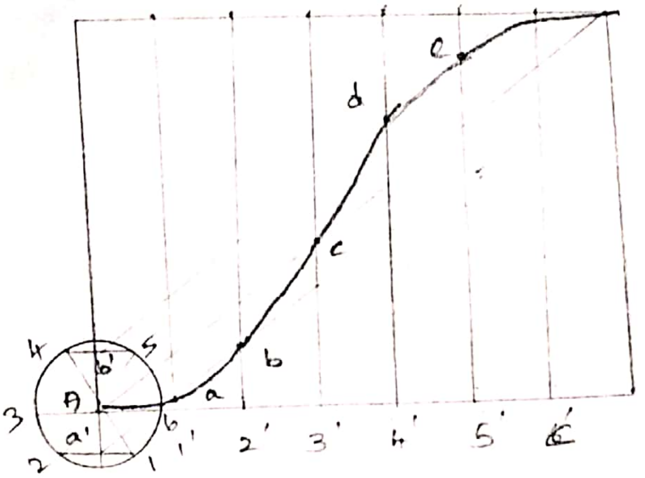
$$V_R = \frac{2\omega S}{\theta_R} \quad , \quad a_R = \frac{4\omega^2 S}{(\theta_R)^2}$$

Scanned by CamScanner

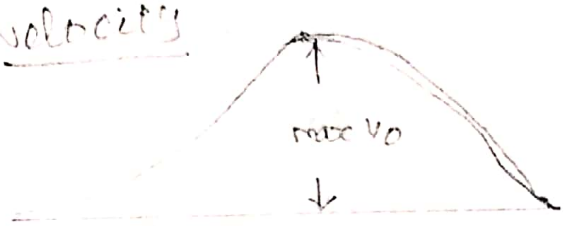
cycloidal motion

$r = S/2\pi$

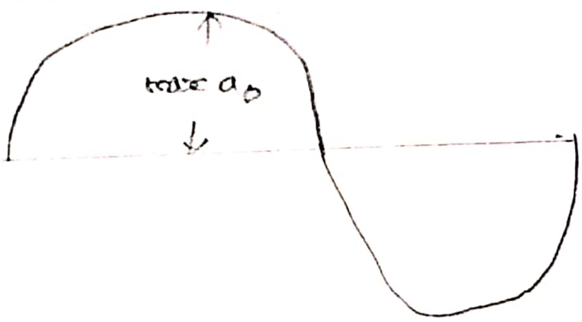
Displacement



velocity



Acceleration



During outstroke

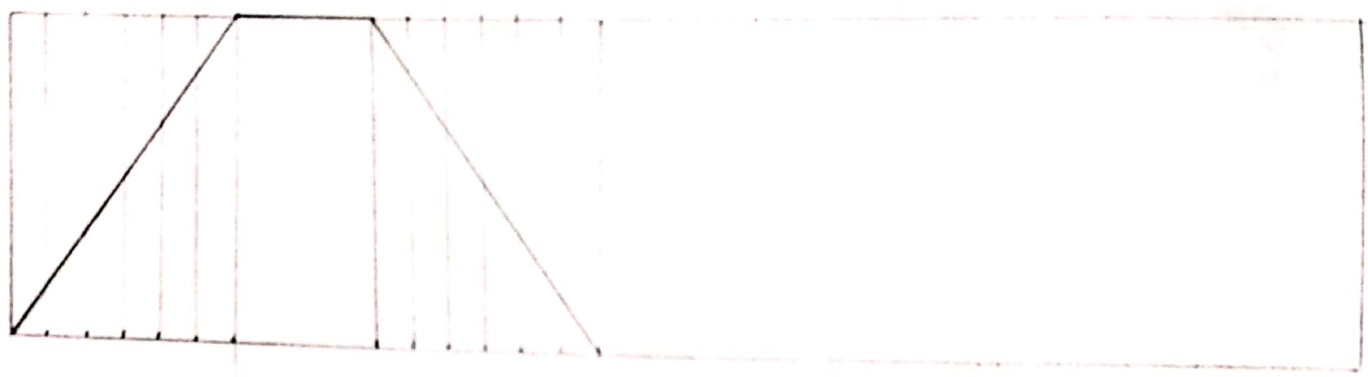
$$V_o = \frac{2\omega S}{\theta_o} \qquad a_o = \frac{2\pi\omega^2 S}{(\theta_o)^2}$$

During return stroke

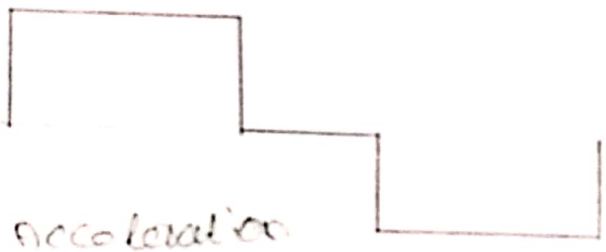
$$V_R = \frac{2\omega S}{\theta_R} \qquad a_R = \frac{2\pi\omega^2 S}{(\theta_R)^2}$$

uniform velocity or uniform motion

displacement



velocity



acceleration

19/8/11

3,4, ~~7~~, 51, 56,

11, ~~18~~, ~~24~~, ~~34~~, ~~37~~, ~~38~~, 39, ~~60~~, ~~61~~,

...n speed cams

- ~~cy~~cloidal motion high speed operations
- SHM low speed operations

cams with specified contours

circular arc cam with flat faced follower

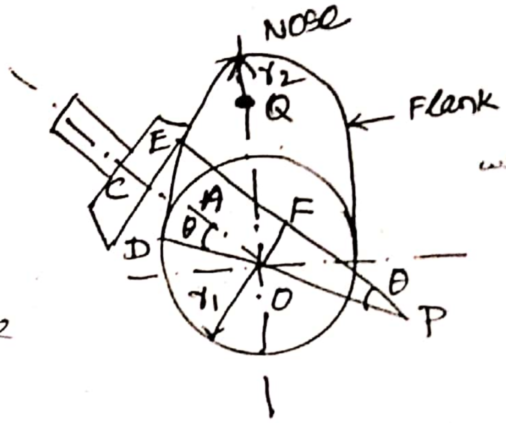
Tangent cam with roller follower

Circular arc cam with flat faced follower

when the flanks of the cam connecting the base circle and nose are of convex circular arc, then the cam is known as circular arc cam.

Assume the cam is fixed and the follower rotates in the opposite direction to that of the cam.

Case 1: when the follower touches the circular flank



O, Q – centres of cam and nose

Let r_1 – least radius of cam or radius of base circle

r_2 – Radius of nose

R – Radius of circular flank = PD = PE

r – Distance between the cam and the nose centres = OQ

when the cam turned through angle θ, the contact point of the flanks with the cam contour has shifted from D to E.

PE is $\perp^{r}$ to face of the follower

OC $\perp^{r}$ to CE, OC $\parallel$ PE

From O draw OF $\perp^{r}$ to PE

The displacement or lift of the follower (x) is given by

$$x = CA$$
$$= OC - OA$$
$$= EF - OA$$
$$= (PE - PF) - OA$$
$$= PE - OP\cos\theta - OA$$
$$= PE - (PD - OD)\cos\theta - OA$$

$x = PE - (PD - OD)\cos\theta - OA$

$= R - (R - r_1)\cos\theta - r_1$

$= R - R\cos\theta + r_1\cos\theta - r_1$

$= R(1 - \cos\theta) - r_1(1 - \cos\theta)$

$$\boxed{x = (R - r_1)(1 - \cos\theta)}$$

velocity of the follower, $v = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$

$x = (R - r_1)(1 - \cos\theta)$

$\frac{dx}{d\theta} = (R - r_1)\sin\theta$

$\therefore v = (R - r_1)\sin\theta \cdot \omega$ $\qquad \because \frac{d\theta}{dt} = \omega$

$$\boxed{v = \omega(R - r_1)\sin\theta}$$

$v_{max} = \omega(R - r_1)\sin\phi \quad (\theta = \phi)$

$\cos\theta \; - \sin\theta$

Acceleration of the follower $a = \frac{dv}{dt}$

$a = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt}$

$\frac{dv}{d\theta} = (\omega(R - r_1) \cdot \cos\theta)$

$a = [\omega(R - r_1)\cos\theta] \cdot \omega$ $\qquad \frac{d\theta}{dt} = \omega$

$$\boxed{a = \omega^2(R - r_1)\cos\theta}$$

$a_{max} = \omega^2(R - r_1) \quad \theta = 0^\circ$

$a_{min} = \omega^2(R - r_1)\cos\phi \quad \theta = \phi$

. when the follower touches the nose

isplacement $x = CA$

$= OC - OA$

$= EF - OA$

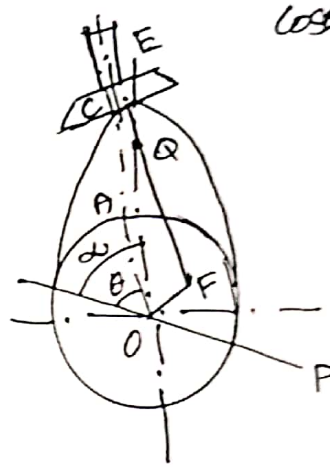
$= QE + QF - OA$

$= QE + OQ\cos(\alpha-\theta) - OA$

$= r_2 + r\cos(\alpha-\theta) - r_1$

$$\boxed{x = r_2 - r_1 + r\cos(\alpha-\theta)}$$

$\cos\theta = -\sin\theta$



velocity $v = \frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dx}{d\theta}\cdot\omega$

$\frac{dx}{d\theta} = r\sin(\alpha-\theta)$

$$\boxed{v = \omega r\sin(\alpha-\theta)}$$

$V_{max} \rightarrow \theta = \infty$

Acceleration $a = \frac{dv}{dt} = \frac{d\theta}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dv}{d\theta}\cdot\omega$

$\frac{dv}{d\theta} = \omega r\cos(\alpha-\theta)(-1)$

$$\boxed{a = -\omega^2 r\cos(\alpha-\theta)}$$

$a_{max} \rightarrow (\alpha-\theta) = 0^\circ$ or $\theta = \infty$

$a_{min} \rightarrow \theta = \phi$

...gent cam with Reciprocating Roller follower

When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as Tangent cam.

O, K – centres of the cam and nose

EU, PQ – straight flanks

r_1 – base radius of cam

r_2 – radius of nose

r_3 – radius of roller

r – Distance b/w the cam and nose centres

α – Angle of ascent

ϕ – Angle turned by the cam for contact of roller with straight flank

θ – Angle turned by the cam from the begining of the roller displacement

ω – Angular velocity of the cam

Case 1: When the roller has contact with straight Flanks

When the cam turns through an angle θ relative to the roller, the centre of the roller is shifted from B to C. the displacement (x) is given by From the geometry, the

$$x = OC - OB$$

$$= \frac{OB}{\cos\theta} - OB = OB\left[\frac{1}{\cos\theta} - 1\right]$$

$$\because OB = r_1 + r_3$$

$$x = (r_1 + r_3)\left[\frac{1}{\cos\theta} - 1\right]$$

velocity of the follower $V = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$

$\sec\theta$

$\sec\theta \cdot \tan\theta$

$\frac{1}{\cos} \cdot \frac{\sin\theta}{\cos\theta}$

$$\frac{dx}{d\theta} = r_1 + r_3\left[\frac{\sin\theta}{\cos^2\theta} - 0\right]$$

$$\boxed{\therefore V = \omega(r_1 + r_3)\frac{\sin\theta}{\cos^2\theta}}$$

When $\theta = \phi$

$V_{max} = \omega(r_1 + r_3)\frac{\sin\phi}{\cos^2\phi}$

When $\theta = 0^\circ$

$V_{min} = 0$

Acceleration of the follower $a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$

$$V = \omega(r_1+r_3)\,\frac{\sin\theta}{\cos^2\theta} \qquad \frac{u}{v}$$

$$\frac{dv}{d\theta} = \omega(r_1+r_3)\left[\frac{\cos^2\theta\cos\theta - \sin\theta(-2\sin\theta\cos\theta)}{\cos^4\theta}\right]$$

$$= \omega(r_1+r_3)\left[\frac{\cos^3\theta + 2\sin^2\theta\cos\theta}{\cos^4\theta}\right]$$

$$= \omega(r_1+r_3)\,\frac{\cos\theta(\cos^2\theta + 2\sin^2\theta)}{\cos^4\theta}$$

$$= \omega(r_1+r_3)\,\frac{\cos^2\theta + 2\sin^2\theta}{\cos^3\theta}$$

$$\therefore\ a = \omega^2(r_1+r_3)\,\frac{2(\cos^2\theta + \sin^2\theta) - \cos^2\theta}{\cos^3\theta}$$

$$\boxed{a = \frac{\omega^2(r_1+r_3)(2-\cos^2\theta)}{\cos^3\theta}}$$

~~a_{min}~~ when $\theta = \phi$

$$a_{max} = \omega^2(r_1+r_3)\,\frac{2-\cos^2\phi}{\cos^3\phi}$$

when $\theta = 0^\circ$

$$a_{min} = \omega^2(r_1+r_3)$$

the roller has contact with nose

AK $\perp^{lr}$ to OD

Displacement $x = OD - OR$

$= (OA + AD) - OB$

θ_1 - Angle turned by the cam measured from the position when the roller is at the top of the nose

$x = OK \cos\theta_1 + DK \cos\theta_1 - OB$

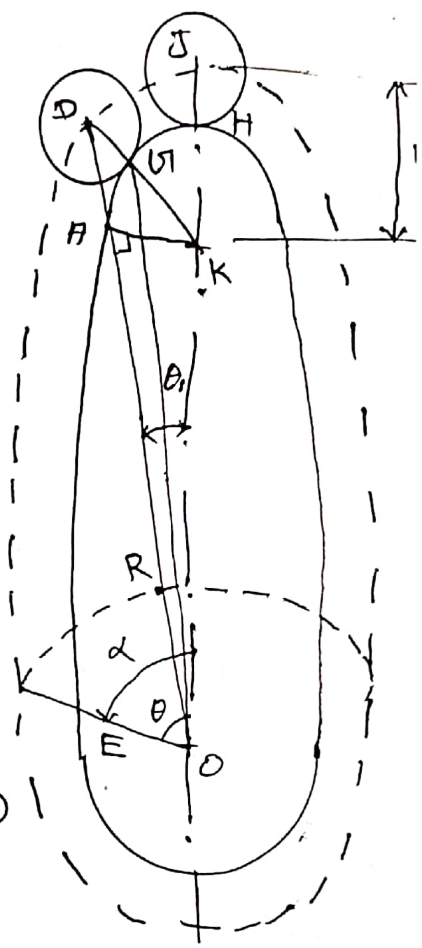
$= OK \cos\theta_1 + DK \sqrt{1-\sin^2\theta_1} - OB$

$= OK \cos\theta_1 + DK \sqrt{1 - \frac{(AK)^2}{(DK)^2}} - OB$

$= r\cos(\alpha-\theta) + (r_2+r_3)\sqrt{1 - \frac{r_2^2}{(r_2+r_3)^2}} - (r_1+r_3)$

$= \cancel{r\cos(\alpha-\theta)} + \cancel{(r_2+r_3)}\left(1 - \cancel{\frac{r_2^2}{r_2+r_3}}\right)$

$\sqrt{1 - \cancel{\frac{r_2^2}{(r_2+r_3)^2}}} = \cancel{r_2^2 + r_3^2 - r_2^3}$



$$\boxed{x = r\cos(\alpha-\theta) + \sqrt{l^2 - r^2\sin^2(\alpha-\theta)} - n}$$

$l = DK = r_2 + r_3$

$n = OB = r_1 + r_3$

2.

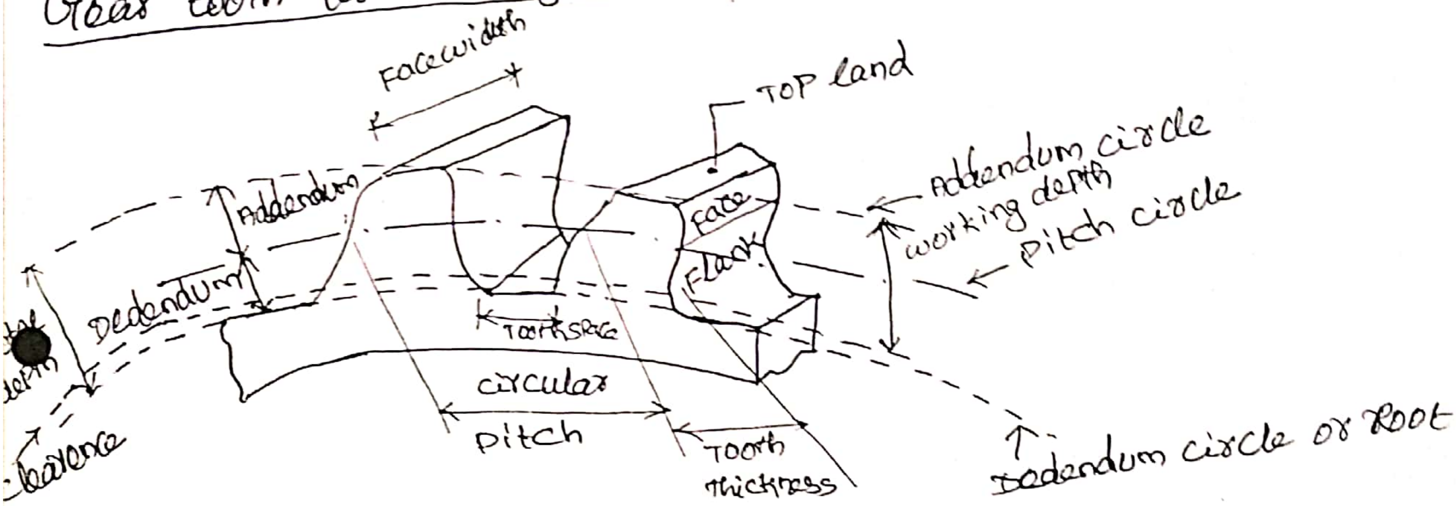
Gears

A gear or cogwheel is a rotating machine part having cut teeth or cogs which mesh with another toothed part to transmit torque.

Classifications of gears

1. According to the position of axes of the shaft
   a. Parallel b. Intersecting c. non parallel and non-intersecting
2. According to the Peripheral velocity of the gears
   a. Low velocity b. Medium velocity c. High velocity
3. According to the type of gearing
   a. Internal b. External c. Rack & pinion
4. According position of the teeth on gear surface
   a. straight b. Inclined c. curved.

Gear tooth terminology



1. pitch circle
2. pitch circle diameter
3. pitch point
4. pitch surface
5. Pressure angle (ϕ) – $14\frac{1}{2}°$ and $20°$
6. Addendum
7. Dedendum
8. Addendum circle
9. Dedendum circle

Merits:
- It transmit exact velocity ratio
- To transmit [illegible]
- It has high efficiency [illegible]
- It has reliable service
- It has [illegible] [illegible]

Demerits
- requires special tools and [illegible] for manufacturing
- The error in cutting teeth [illegible] may cause vibrations [illegible]

10. circular pitch

$P_c = \pi D/T$

D - dia of pitch circle

T - Number of teeth on the wheel

11. Diametral pitch (P_d)

$P_d = \frac{T}{D} = \frac{\pi}{P_c}$ $\left[\because P_c = \pi D/T\right]$

12. module pitch or module (m)

$m = D/T$

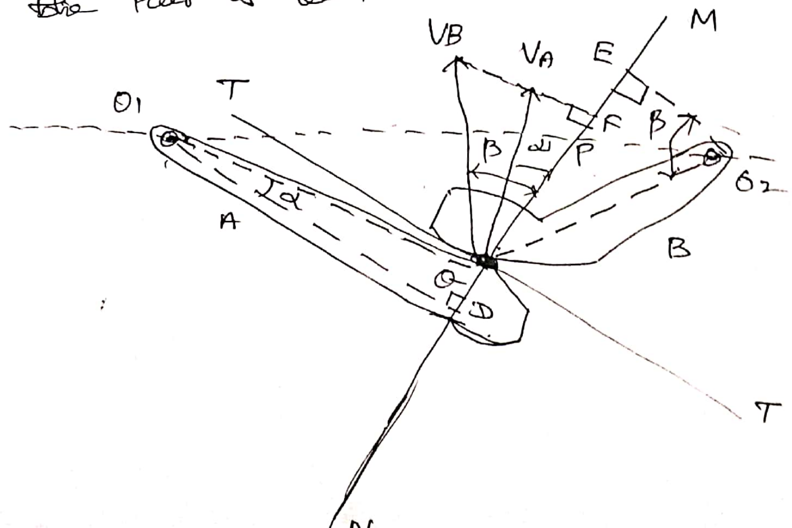
Gear materials

metallic gears - cast iron, steel and bronze

Non metallic gears - Raw hide, wood, compressed paper and synthetic resin like nylon,.. etc.

Law of Gearing

For obtaining constant velocity ratio, at any instant of teeth the common normal at the point of contact between the pair of teeth must always passes through the pitch point



$V_A = \omega_A \times O_1Q$

$V_B = \omega_B \times O_2Q$

Component of V_A along common normal = component of V_B along common normal

$$V_A \cos\alpha = V_B \cos\beta \rightarrow (1)$$

$$\omega_A O_1Q \cos\alpha = \omega_B (O_2Q) \cos\beta$$

~~$\omega_A O_1Q$~~ From the $\Delta\, Q D Q$ & $\Delta\, O_2EQ$

$$\cos\alpha = \frac{O_1D}{O_1Q} \qquad \cos\beta = \frac{O_2E}{O_2Q}$$

$$\omega_A\, O_1Q \frac{O_1D}{O_1Q} = \omega_B\, O_2Q \frac{O_2E}{O_2Q}$$

$$\omega_A\, O_1D = \omega_B\, O_2E$$

$$\frac{\omega_A}{\omega_B} = \frac{O_2E}{O_1D} \rightarrow (2)$$

But from ΔO_1DP & ΔO_2EP

$$\cancel{\frac{O_1E}{O_2D}} \quad \frac{O_2E}{O_1D} = \frac{O_2P}{O_1P} \rightarrow (3)$$

From the above eqn (3) we proove the law of gearing.

Form of gear tooth profile

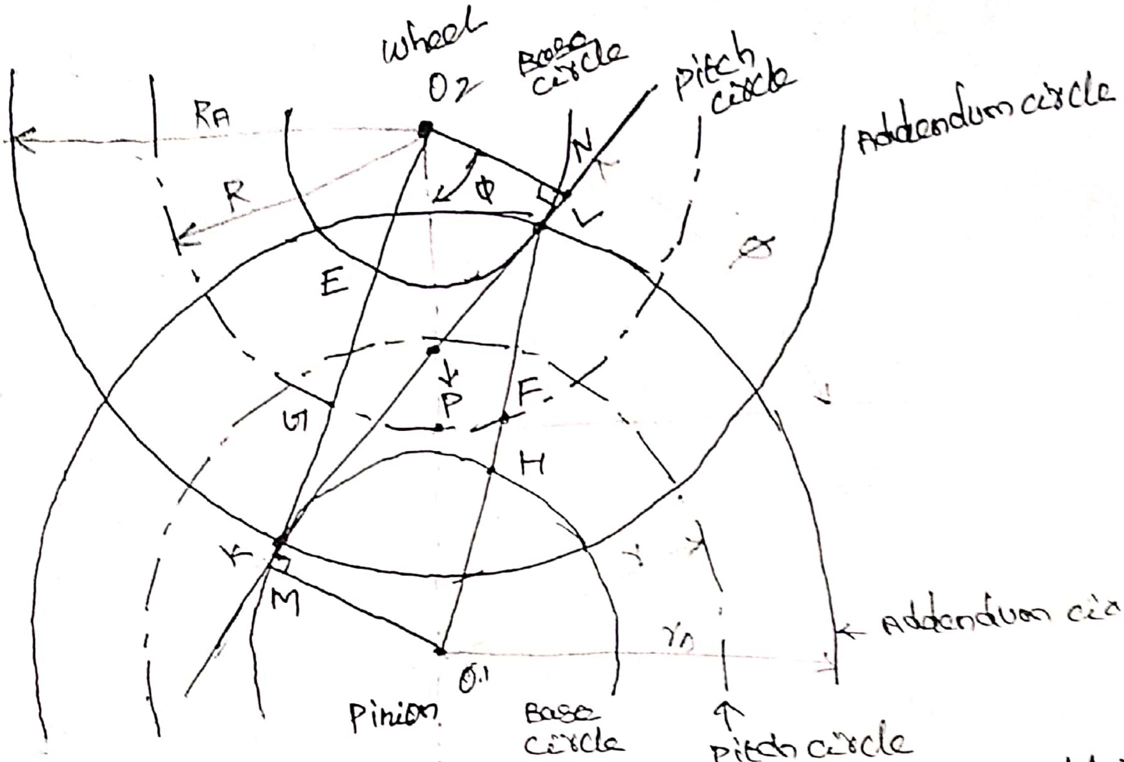
1. cycloidal tooth profile
2. Involute tooth profile

- comparison between Involute and cycloidal tooth profile

Involute	Cycloidal
1. variation in centre distance does not affect the velocity ratio	centre distance should not vary
2. pressure angle remains constant	pressure angle varies
3. Interference occurs	No interference occurs
4. Easy to manufacture	Dificult to manufacture
5. weaker teeth	stronger teeth

Length of Path of contact

The length of the common normal cut off by the addendum circles of the wheel and pinion.



Two involute gears pinion and wheel in mesh. Pinion rotates in cw direction. The contact btn the teeth begins at K and ends at L. The length of path of contact is KL.

Point K is located on the flank near the base circle of pinion.

Point L is located on the flank near the base circle of wheel. MN common Tangent.

$KL = KP + PL$

KL – Path of contact

KP – path of approach

PL – Path of recess

$r = O_1P$ = Radius of pitch circle of pinion

$r_A = O_1L$ = Radius of addendum circle of pinion

$R = O_2P$ = Radius of pitch ~~pitch~~ " wheel

$R_A = O_2K$ = " addendum circle of wheel adius of Base circle of pinion is given by

$O_1M = O_1P\cos\phi$

$O_1M = r\cos\phi$

The radius of Base circle of wheel is given by

$O_2N = O_2P\cos\phi$

$= R\cos\phi$

The length path of approach $KP = KN - PN$

The length path of recess $PL = ML - MP$

ΔO_2KN

$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{R_A^2 - R^2\cos^2\phi}$

$PN = O_2P\sin\phi$

$PN = R\sin\phi$

ΔO_1ML

$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{r_A^2 - r^2\cos^2\phi}$

$MP = O_1P\sin\phi = r\sin\phi$

$\therefore KL = KP + PL$

$$\boxed{KL = \sqrt{R_A^2 - R^2\cos^2\phi} + \sqrt{r_A^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

Length of arc of contact

The length of arc made by a point on the pitch circle from begining to the end of engagement of both pair.

$$\boxed{\text{Length of arc of contact} = \frac{\text{length of path of contact}}{\cos\phi}}$$

Contact ratio $= \frac{\text{Length of arc of contact}}{\text{circular pitch } (P_c)}$

$P_c = \pi \, D/T = \pi \cdot m$

m - module

Pitch circle radius of gear $R = \frac{m \cdot T_G}{2}$

Pitch circle radius of pinion $r = \frac{m T_P}{2}$

Addendum radius of gear wheel

$R_A = R + \text{adde}$

Addendum radius of pinion

$r_A = r + \text{addend}$

Interference

The phenomenon when the tip of tooth undercuts the root on it mating gear is known as interference.

methods to avoid interference

1. The height of the teeth may be reduced
2. The pressure angle may be increased

under cutting

The radial flank of the pinion may be ~~to~~ cut back.

$$\text{Gear ratio} = \frac{T_G}{T_P} = \frac{R}{r} = \frac{\omega_P}{\omega_G}$$

Minimum number of teeth on the pinion to avoid interference

$$T_P = \frac{2 A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

$A_P \rightarrow$ Addendum of pinion $(A_P \cdot m)$

minimum number of teeth on the wheel to avoid interference

$$T_G = \frac{2 A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$A_W \rightarrow$ Addendum of the wheel $(A_W \cdot m)$

maximum velocity of sliding at engagement $= (\omega_P + \omega_G) KP$

maximum velocity of sliding at disengagement $= (\omega_P + \omega_G) PL$

Gear Trains

...wo or more gears are made to mesh with each other to ...smit power from one shaft to another. Such combination is ...lled gear train.

Types of gear train

- Simple
- Compound
- Reverted
- Epicyclic

Simple Gear train

- velocity ratio $= \frac{\text{speed of driver}}{\text{speed of driven}} = \frac{\text{No of teeth on driven}}{\text{No of teeth on driver}}$

(or)

speed ratio $= \boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}}$

Train value $= \frac{1}{\text{speed ratio}}$

Compound Gear Train

- more than one gear on a shaft

- speed ratio $= \frac{\text{speed of the first driver}}{\text{speed of the last driven}}$

$= \frac{\text{product of no of teeth on the drivens}}{\text{product of no of teeth on the drivers}}$

$\therefore \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$

merits

- Larger speed ratio
- motion can be transmitted in the round bends and corners
- Different types of gears may be used in order to suit the given conditions.

m

Non-standard gears

The gear teeth obtained by modifying the standard of gear teeth parameters, is known as non-standard gears.

Reason

- To eliminate undercutting
- To prevent interference
- To maintain a reasonable contact ratio.

Advantages - ?

Helical gears → modification of ordinary spur gear. - reduce noise.

Herringbone gears → double helical gear → avoid axial thrust of single helical gear.

Spiral gears.

Bevel gears → Two shaft whose axes are intersecting.

worm and worm gear

Rack and pinion.

gear train

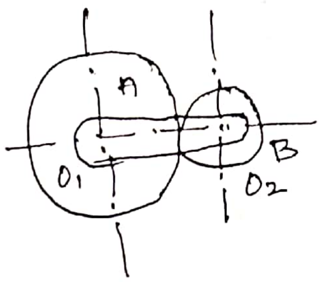
when the axes of the first gear and the last gear are co-axial then the gear train is known as reverted gear train

Application

- Automobile gear boxes
- Lathe back gears
- speed reducers
- clocks

Epicyclic Gear Train (or) planetary gear train

when the gears are arranged in such a manner that one or more gears move upon and around another gear, then the gear train is known as Epicyclic gear train.



Applications:-

- Differential gears
- Back gear of lathe
- wrists watches
- pulley blocks
- Hoists

Types of Epicyclic gear Train

- simple Epicyclic Gear Train
- compound Epicyclic Gear Train

velocity ratio of Epicyclic gear Train

1. Tabular method
2. Algebraic method.

~~9, 16, 19, 21, 24, 28, 29, 32, 34, 38, 40, 44, 45~~

Table of motions

Step No	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed. gear A rotates through +1 revolution ie, 1 rev. anticlockwise	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed. gear A rotates through +x revolutions	0	+x	$-x\frac{T_A}{T_B}$
3.	Add +y revolutions to all elements	+y	+y	+y
4	Total motion	+y	x+y	$y-x\frac{T_A}{T_B}$

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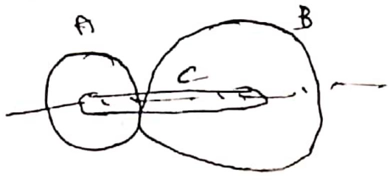
In an epicyclic gear train an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rpm in the ccw direction about the centre of the gear A which is fixed, determine the speed of the gear B. If the gear A instead of being fixed, makes 300 rpm in the clockwise direction, what will be the speed of gear B?



Given: $T_A = 36$, $T_B = 45$, $N_C = 150$ rpm

$y = +150$

Gear A is fixed. $x+y = 0 \Rightarrow x + 150 = 0$

$x = -150$ rpm

speed of gear B: $N_B = y - x\frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = 270$ rpm (ccw)

speed of gear B when gear A makes 300 rpm clockwise

$x+y = -300$

$x = -300 - y \Rightarrow x = -300 - 150 = -450$ rpm

$N_B = y - x\frac{T_A}{T_B} = 150 + 450 \frac{36}{45} = 510$ rpm (ccw)

1. Classifications of gears

2. Gear tooth terminology

circular pitch $P_c = \frac{\pi D}{T}$

D - dia of the pitch circle

T - Number of teeth on the wheel.

Diametral pitch $P_d = \frac{T}{D}$

$P_d = \frac{\pi}{P_c}$ $\left[P_c = \frac{\pi D}{T}\right]$

module $m = \frac{D}{T}$

3. Length of path of contact

It is the length of the common normal cut-off by the addendum circles of the wheel & pinion.

4. Length of arc of contact

It is the path traced by the point on the pitch circle from the begining to the end of engagement of a given pair of teeth. The arc of contact consists of two parts.

a). Arc of approach: It is the portion of the path of contact from the begining of the engagement to pitch point.

b) Arc of recess:- It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

- Contact ratio = $\frac{\text{length of arc of contact}}{\text{circular pitch}}$

- Gear materials – metallic or non metallic

  Cast iron
  Steel
  Bronze

  wood
  raw hide
  compressed paper
  Synthetic resin
  } reduced noise

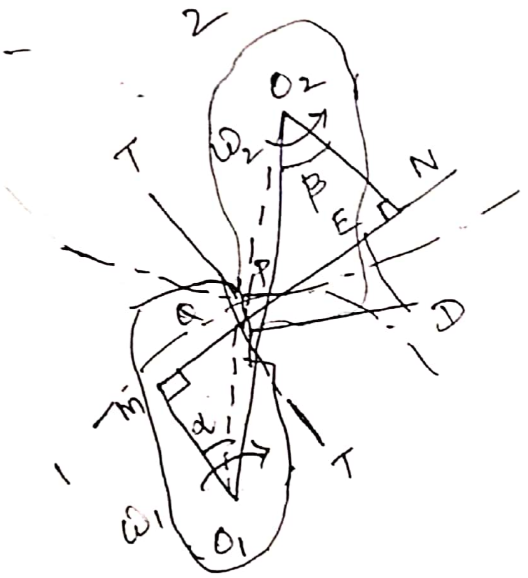
- Condition for constant velocity Ratio of toothed wheels

(or)

Law of Gearing

The common normal at the point of contact between pair of teeth must always passes through the pitch point.

reff :-



- Consider the portions of the two teeth
- Let TT be the common tangent and MN be the common normal at the point of contact Q.
- From the centres O_1 and O_2 draw O_1M and O_2N ⊥ler to MN
- Let V_1 and V_2 be the velocities of the point Q on the wheel 1 and 2.

$$V_1 \cos\alpha = V_2 \cos\beta$$

$$(\omega_1 \times O_1Q)\cos\alpha = (\omega_2 \times O_2Q)\cos\beta$$

$$\omega_1 \times O_1Q\left[\frac{O_1M}{O_1Q}\right] = \omega_2 \times O_2Q\left[\frac{O_2N}{O_2Q}\right]$$

$$\omega_1 \times O_1M = \omega_2 \times O_2N$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \rightarrow (i)$$

the similar triangles O_1MP & O_2NP

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \rightarrow (2)$$

(1) = (2)

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \rightarrow (3)$$

∴ we proved the law of gearing.

Forms of teeth

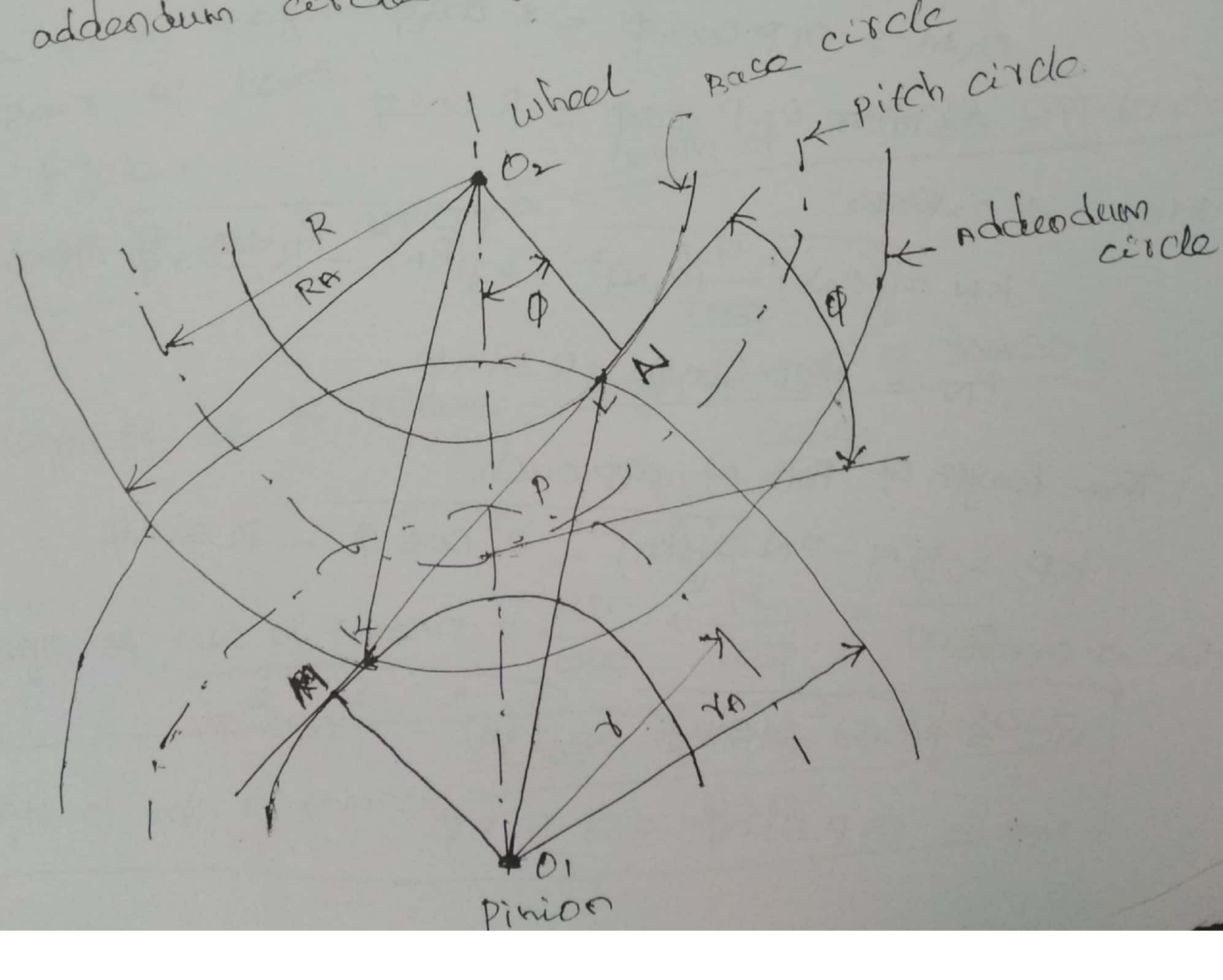
1. cycloidal teeth
2. Involute teeth.

System of gear teeth

1. $14\frac{1}{2}^\circ$ composite system
2. $14\frac{1}{2}^\circ$ full depth involute system
3. 20° full depth involute system
4. 20° stub involute system.

Length of path of contact

It is the length of the common normal cut off by the addendum circles of the wheel and pinion.



Consider two involute gears i.e pinion and wheel in mesh. When the pinion rotates in clockwise direction, the contact between a pair of teeth begins at point K and ends at point L. Therefore the length of path of contact is KL.

The length of KP is known as path of approach
The length of PL is known as path of recess

Let

r = Radius of the pitch circle of the pinion = O_1P
r_A = Radius of addendum circles of pinion = O_1L
R = Radius of pitch circle of wheel = O_2P
R_A = Radius of addendum circles of wheel = O_2K

From the figure

$$O_1M = O_1P\cos\phi = r\cos\phi$$

$$O_2N = O_2P\cos\phi = R\cos\phi$$

From ΔO_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

∴ The length of path of approach

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

From ΔO_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

of path of recess

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ length of path of contact,

$$KL = KP + PL$$

$$= \left[\sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi\right] + \sqrt{(r_A)^2 - r^2\cos^2\phi} - r$$

$$\boxed{KL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

Length of arc of contact

It is the path traced by a point on the pitch circle from begining to end of the engagement of the pair of teeth.

From figure,

The length of arc of approach $= \dfrac{\text{length of patch of approach}}{\cos\phi}$

$$= \frac{KP}{\cos\phi}$$

The length of arc of recess $= \dfrac{\text{length of path of recess}}{\cos\phi}$

$$= \frac{PL}{\cos\phi}$$

The length of arc of contact $= \dfrac{KP}{\cos\phi} + \dfrac{PC}{\cos\phi} = \dfrac{KL}{\cos\phi}$

$$\boxed{\text{Length of arc of contace} = \frac{\text{Length of path of contact}}{\cos\phi}}$$

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Angle turned through by pinion

$$= \frac{\text{Length of arc of contact}}{\text{circumference of pinion}} \times 360^\circ$$

Pitch circle Radius of gear, $R = \frac{m \cdot T}{2}$

Pitch circle radius of pinion, $r = \frac{m \cdot t}{2}$

where,

m - module

T - Number of teeth on gear

t - Number of teeth on pinion

Addendum Radius of gear wheel $R_A = R +$ Addendum

Addendum radius of pinion $r_A = r +$ addendum

Interference

The phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.

minimum number of teeth on the pinion in order to avoid interferen

$$t = \frac{2A_p}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

where, A_p - Addendum of the pinion

G - Gear ratio $(G = T/t)$

ϕ - pressure angle or angle of obliquity

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$$A_p = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

Minimum number of teeth on the wheel in order to avoid interfere

$$T = \frac{2A_w}{\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi}-1} = \frac{2A_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$A_w = \frac{T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi}-1\right]$$

where,

A_w - Addendum of wheel

Unit-IV: Gears, Part-B Theory Questions

1. Drive an expression for the minimum number required for the pinion in order to avoid interfe

2. How are gears classified?

3. Drive the expression for velocity ratio of a simple gear train. (Law of gearing)

4. What do you mean by pitch point, circular pitch, module, addendum and pressure angle? Explain with neat sketch

5. Drive an expression for minimum number of teeth on wheel in order to avoid interference.

6. State and prove the law of gearing.

7. Derive the expression to determine the length of path of contact of meshing gear teeth.

8. Draw a bevel gear automotive differential and explai its principle of working.

9. What is meant by interference in gears? What are the measures to eliminate the same.

10. Briefly explain the sub-classification of compound gear trains with neat sketches.

11. Explain the procedure adopted for designing the spur gears.

minimum number of teeth on the pinion to avoid interference.

Let, t - No. of teeth on pinion

T - No. of teeth on wheel

m - module of teeth

r - pitch circle radius of pinion $mt/2$

G - Gear ratio $\left(T/t = R/r\right)$

ϕ - pressure angle

From triangle O_1NP

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2\sin^2\phi - 2r \cdot R\sin\phi \cos(90+\phi)$$

$\therefore$ Limiting radius of the pinion addendum circle

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi}$$

$$= \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}$$

Let $A_p \cdot m$ - Addendum of pinion. Where A_p is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

WKT addendum of pinion $O_1N - O_1P$ $\quad O_1P = r = mt/2$

$$A_p \cdot m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{mt}{2}$$

$$= \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$A_p = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} \ldots\right.$$

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$$t = \frac{2Ap}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1}$$

$$t = \frac{2Ap}{\sqrt{1 + \sigma\left(\sigma+2\right)\sin^2\phi} - 1}$$

22, 16, 32, 45, 48 → note

ans – 19, 20

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... having 24 teeth drives a gear having 60 teeth. ... profile of the gears are involute with 20° pressure angle, 10mm module and 10mm addendum. Find the length of path of contact, arc of contact and contact ratio.

Given data:-

$T = 60$

$t = 24$

$\phi = 20^\circ$

$m = 10\,mm$

addendum $= 10mm$

To find:-

i). Length of path of contact ii). Length of arc of contact iii). contact ratio.

Solution.

WKT

Length of path of contact $KL = KP + PL$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$\text{Length of arc of contact} = \frac{KL}{\cos\phi}$$

$$\text{contact ratio} = \frac{\text{length of arc of contact}}{\text{circular pitch }(P_C)} \qquad \because P_C = \pi m.$$

$R_A = R + \text{addendum}$ $\quad R = \frac{mT}{2} = \frac{10\times60}{2} = 300m$

$r_A = r + \text{addendum}$ $\quad r = \frac{mt}{2} = \frac{10\times24}{2} = 120$

$R_A = 300 + 10 = 310\,mm$

$r_A = 120 + 10 = 130\,mm$

$$\therefore KP = \sqrt{310^2 - 300^2\cos^2(20^\circ)} - 300\times\sin 20^\circ$$

$= 102.60$

$$PL = \sqrt{130^2 - 120^2 \cos^2 20^\circ} - 120 \sin 20$$

$$= 64.68 - 41.04$$

$$= 23.64 \text{ mm}$$

$$KL = 26.34 + 23.64 = 49.98 \text{ mm}$$

$$\text{length of arc of contact} = \frac{49.98}{\cos 20^\circ} = 53.18 \text{ mm}$$

Contact ratio = $\frac{53.18}{\pi \times 10} = 1.69$ say 2

Number of pairs of teeth in contact

Result:-

1). Length of Path of contact $KL = 49.98$ mm

2). Length of arc of contact $= 53.18$ mm

3). contact ratio $= 2$

(2). Angle turned through by the pinion

$$\frac{\text{Length of arc of contact} \times 360}{\text{circumference of Pinion } (2\pi r)}$$

Angle turned through by the gear

$$\frac{\text{length of arc of contact} \times 360}{\text{circumference of Gear} (2\pi R)}$$

$\omega_1 =$ pinion, $\omega_2 =$ wheel

$$\omega_1 / \omega_2 = T/t \Rightarrow \omega_2 = \omega_1 \times t/T$$

$$V_R = \omega_1 \cdot r = \omega_2 \cdot R$$

$$V_S = (\omega_1 + \omega_2)\, KP$$

$$V_S / V_R = ?$$

$$V_S = (\omega_1 + \omega_2)\, PL$$

$$V_S / V_R = ?$$

5 Kopy

Unit - IV : Gears

Part - B Problems

1. Two gear wheels mesh externally to give a velocity ratio of 3 to 1. The involute teeth has 6mm module and 20° pressure angle. Addendum equal to one module. Determine the number of teeth on pinion to avoid interference and the corresponding number on the wheel.

Given data:

Velocity ratio (or) Gear ratio $G = 3$

module $m = 6mm$

pressure angle $\phi = 20°$

(A) Addendum $= 1$ module $= 6mm$

To find:-

The number of teeth required in order to avoid interference on 1). pinion 2). wheel

Solution:-

W.K.T.

$$t = \frac{2\,A_P}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$$

$$= \frac{2\times 6}{\sqrt{1+3(3+2)\sin^2(20°)} - 1}$$

$= 18.12$ say 19

W.K.T $\quad G = \frac{T}{t}$

$$3 = \frac{T}{19} \Rightarrow T = 3\times 19$$

$T = 57$

Result:-

The number of teeth required in order to avoid interference on

1). Pinion = 19 teeth

2). Wheel = 57 teeth

—x—

(2). Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are involute form; module = 6mm, addendum = one module, pressure angle = 20°, The pinion rotates at 90 rpm. Find (1). number of teeth on pinion to avoid interference, on it and the corresponding number on the wheel. 2). Length of path of contact 3). number of pairs of teeth in contact. 4). The maximum velocity of sliding

Given data:-

velocity ratio or gear ratio $G = 3$

module (m) = 6mm

addendum = 1 module = 6mm

pressure angle $\phi = 20°$

Speed of pinion $N_1 = 90$ rpm

To find:-

1). Number of teeth on pinion to avoid interference

2). Number of teeth on wheel to avoid interference

3). Length of path of contact

4). number of pairs of teeth in contact

5). The maximum velocity of sliding

Solution:-

Number of teeth on pinion to avoid interference

$$t = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

of teeth on wheel to avoid interference

$$T = \frac{2 A_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

Length of path of contact $KL = KP + PL$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$KL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

Number of pairs of teeth in contact (or) contact ratio $= \dfrac{\text{Length of arc of contact}}{\text{circular pitch}}$

- maximum velocity of sliding $V_s = (\omega_1 + \omega_2)\,KP$

$$= \frac{2 A_p}{\sqrt{1+G(G+2)\sin^2\phi} - 1} \Rightarrow t = \frac{2\times 6}{\sqrt{1+3(3+2)\sin^2 20^\circ} - 1} = \frac{12}{\sqrt{2.755} - 1} = \frac{12}{0.659}$$

$t = 18.19$ say 19

$$\boxed{t = 19}$$

- WKT $G = T/t \Rightarrow 3 = \dfrac{T}{19} \Rightarrow 19 \times 3 = T$

$$\boxed{T = 57}$$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

~~Pitch circle~~

Radius of addendum circle of wheel $R_A = R +$ Addendum

Pitch circle radius of wheel $\Big\}$ $R = \dfrac{M.T}{2} = \dfrac{6\times 57}{2} = 171$ mm

$\therefore R_A = R +$ Addendum

$= 171 + 6 = 177$ mm

$$\boxed{R_A = 177\text{ mm}}$$

Radius of addendum circle of pinion $(r_A) = r + a$

Radius of pitch circle of pinion $r = \frac{m \cdot t}{2} = \frac{6 \times 19}{2} =$

$r_A = r + \text{addendum} = 57 + 6 =$

$\boxed{r_A = 63 \text{ mm}}$

$\therefore KP = \sqrt{R_A^2 - R^2 \cos^2\phi} - R \sin\phi$

$= \sqrt{177^2 - 171^2 \cos^2 20^\circ} - 171 \sin 20^\circ$

$= \sqrt{31329 - 29241 (\cos 20^\circ)^2} - 58.485$

$= 74.219 - 58.485 = 15.734 \text{ mm}$

$\boxed{KP = 15.734 \text{ mm}}$

$PL = \sqrt{r_A^2 - r^2 \cos^2\phi} \cancel{+ r\sin\phi} - r\sin\phi$

$= \sqrt{63^2 - 57^2 \cos^2 20^\circ} - 57 \sin 20^\circ$

$= 33.167 - 19.495 = 13.671 \text{ mm}$

$\boxed{PL = 13.671 \text{ mm}}$

$KL = KP + PL = 15.734 + 13.671 = 29.405 \text{ mm}$

$\boxed{KL = 29.405 \text{ mm}}$

Length of arc of contact $= \frac{\text{Length of Path of Contact } (KL)}{\cos\phi}$

$= \frac{29.405}{\cos 20^\circ} = 31.292 \text{ mm}$

circular pitch $p_c = \pi m$

$= \pi \times 6 = 18.852 \text{ mm}$

$\boxed{p_c = 18.852 \text{ mm}}$

of pairs of teeth in contact = $\frac{\text{Length of arc of contact}}{\text{circular pitch}}$

$$= \frac{31.292}{18.852} = 1.659$$

$$= 1.659 \text{ say } 2$$

$$\boxed{\text{Number of pairs of teeth in contact} = 2}$$

$$V_s = (\omega_1 + \omega_2)\, KP$$

WKT $\frac{\omega_1}{\omega_2} = \frac{T}{t}$, $\omega_1 = \frac{2\pi N_1}{60} = \frac{2\times\pi\times 90}{60}$

$$\omega_1 = \cancel{3.14}\ 9.424 \text{ rad/s}$$

$$\omega_2 = \omega_1 \times t/T$$

$$= \cancel{3.14}\ 9.424 \times \frac{19}{57}$$

$$= 3.141 \text{ rad/s}$$

$$V_s = (9.424 + 3.141)\, 15.734 = 197.697 \text{ mm/s}$$

$$\boxed{V_s = 197.697 \text{ mm/s}}$$

Result:-

1). Number of teeth on pinion to avoid interference = 19

2). Number of teeth on wheel to avoid interference = 57

3). Length of path of contact = 29.405 mm

4). Number of pairs of teeth in contact = 2

5). The maximum velocity of sliding = 197.697 mm/s

— x —

(3). Two gear wheels mesh externally and ... velocity ratio of 3. The teeth are of involute ... module 6 mm. The standard addendum is ... If the pressure angle is 18°, and pinion rotates ... find i). Number of teeth on each wheel so that ... is just avoided

ii). The length of path of contact

iii). maximum velocity of sliding

Note :- This problem is similar to problem NO: 2. only the pressure angle is vary.

—x—

(4). A pinion with 20 teeth and 125 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6.25 mm. what is the least pressure angle which can be used to avoid interference? with this pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at a time

Given data :-

$t = 20$

$d = 125\,mm \Rightarrow r = d/2 = 62.5\,mm = OP$

Addendum of pinion $= 6.25\,mm$

Addendum of rack $= 6.25\,mm = LH$

To find :-

1). Least pressure angle (ϕ)

2). Length of arc of contact

3). minimum number of teeth in contact

solution :-

W.K.T. $LH = r \sin^2\phi$

$$\sin^2\phi = \frac{LH}{r}$$

$P_c = 10 \cdot ...$

$$\sin^2\phi = \frac{LH}{r}$$

$$\sin\phi = \sqrt{LH/r} \Rightarrow \phi = \sin^{-1}\sqrt{LH/r}$$

Length of arc of contact = $\frac{\text{length of path of contact}(KL)}{\cos\phi}$

The number of pairs of teeth in contact = $\frac{\text{length of arc of contact}}{\text{circular pitch}}$

$$\phi = \sin^{-1}\sqrt{LH/r} = \sin^{-1}\sqrt{\frac{6.25}{62.5}}$$

$$\phi = \sin^{-1}(0.3162) = 18.434^\circ$$

$$\boxed{\phi = 18.434^\circ}$$

$$KL = \sqrt{(r+LH)^2 - (OP\cos\phi)^2}$$

$$= \sqrt{(62.5+6.25)^2 - (62.5\cos 18.434^\circ)^2}$$

$$= \sqrt{4726.562 - 3515.663} = 34.797\,mm$$

$$\boxed{KL = 34.797\,mm}$$

Length of arc of contact = $\frac{KL}{\cos\phi} = \frac{34.797}{\cos 18.434^\circ} = 36.679\,mm$

$$\boxed{\text{Length of arc of contact} = 36.679\,mm}$$

circular pitch $= \pi m = \frac{\pi D}{T}$ $\quad \because m = D/T$

$$= \frac{\pi \times 125}{20} = 19.634\,mm$$

Number of pairs of teeth in contact = $\frac{\text{length of arc of contact}}{\text{circular pitch}}$

$$= \frac{36.679}{19.634} = 1.868 \text{ say } 2$$

$$\boxed{\text{Number of pairs of teeth in contact} = 2}$$

Result:-

1). Least pressure angle (ϕ) = $18.434°$

2). Length of arc of contact = 36.679 mm

3). minimum number of pairs of teeth in contact = 2

If u want more detail about this problem, please refer Theory of machines by R.S. Khurmi page no: 415, 12.22

— x —

5). A pair of 20° full depth involute spur gears having 30 and 50 teeth respectively of module 4 mm are in mesh. The smaller gear rotates at 1000 rpm. Determine i). sliding velocities at engagement and at disengagement of pair of a teeth and (ii). contact ratio.

Given data:-

pressure angle $\phi = 20°$

number of Teeth on Pinion $t = 30$ (smaller gear)

number of teeth on wheel $T = 50$ (Larger gear)

module $m = 4$ mm

speed $N_1 = 1000$ rpm

To find:-

1). sliding velocity at the engagement

2). sliding velocity at the disengagement

3). contact ratio

solution:-

sliding velocity at the engagement $= (\omega_1 + \omega_2) KP$

sliding velocity at the disengagement $= (\omega_1 + \omega_2) PL$

$$\text{contact ratio} = \frac{\text{length of arc of contact}}{\text{circular pitch}}$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\times\pi\times1000}{60} = 104.72 \text{ rad/sec}$$

$$\frac{\omega_1}{\omega_2} = \frac{T}{t} \Rightarrow \omega_2 = \omega_1 \times \frac{t}{T}$$ (5)

$$\omega_2 = 104.72 \times \frac{30}{50} = 62.83 \text{ rad/sec}$$

$$\therefore KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

Radius of addendum circle of larger gear $R_A = R +$ addendum of larger gear

pitch circle radius of the larger gear $R = \frac{m \cdot T}{2} = \frac{4 \times 50}{2} = 100$ mm

$$\boxed{R = 100 \text{ mm}}$$

Addendum of larger gear (A_w)

$$\text{WKT} \quad T = \frac{2A_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$A_w = \frac{m \cdot T}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right] \quad \left\{ \because G = \frac{T}{t}, \quad \frac{1}{G} = \frac{t}{T} \right.$$

$$= \frac{4 \times 50}{2}\left[\sqrt{1 + \frac{30}{50}\left(\frac{30}{50}+2\right)\sin^2 20^\circ} - 1\right]$$

$$= 100\left[\sqrt{1 + 0.6\,(0.6+2)\,(0.342)^2} - 1\right]$$

$$= 100\left[\sqrt{1 + 0.1824} - 1\right] = 100 \times 0.0874$$

$$= \cancel{100}\ 8.74 \text{ mm}$$

$$\boxed{A_w = 8.74 \text{ mm}}$$

$$R_A = R + \text{addendum} = 100 + 8.74 = 108.74 \text{ mm}$$

$$\boxed{R_A = 108.74 \text{ mm}}$$

$$\therefore KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{108.74^2 - 100^2\cos^2 20^\circ} - 100\sin 20^\circ$$

$= \sqrt{\cancel{2427.46}\ 2994.16} - 34.20 \quad = \cancel{49.27}\ 54.72 - 3$

$\boxed{KP = 20.52\,mm}$

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

Radius of addendum circle of smaller gear $r_A = r +$ addendum

pitch circle radius of smaller gear $r = \frac{m \cdot t}{2} = \frac{4 \times 30}{2} = 60mm$

$\boxed{r = 60mm}$

Addendum of smaller gear (AP)

wkt $\quad t = \frac{2AP}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$

$AP = \frac{m \cdot t}{2}\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right] \quad \because G = T/t$

$= \frac{4 \times 30}{2}\left[\sqrt{1 + \frac{50}{30}\left(\frac{50}{30} + 2\right)\sin^2 20^\circ} - 1\right]$

$= 60\left[\sqrt{1 + 1.67(1.67 + 2)(\sin 20^\circ)^2} - 1\right]$

$= \cancel{60\sqrt{1.522} - 1}$

$= 60\left[\sqrt{1.717} - 1\right] = 18.62\,mm$

$\boxed{AP = 18.62\,mm}$

$r_A = r +$ addendum $= 60 + 18.62 = 78.62\,mm$

$\boxed{r_A = 78.62\,mm}$

$\therefore PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{78.62^2 - 60^2\cos^2 20^\circ} - 60\sin 20$

$$= \sqrt{6181.10 - 3600(0.939)^2} - 20.52$$

$$= \sqrt{3006.90} - 20.52 = 54.84 - 20.52$$

$$\boxed{PL = 34.32 \text{ mm}}$$

sliding velocity at the engagement $= (\omega_1 + \omega_2)\, KP$

$$= (104.72 + 62.83)\, 20.52$$

$$= 3438.13 \text{ mm/s}$$

$$= 3.44 \text{ m/s}$$

$$\boxed{\text{sliding velocity at the engagement} = 3.44 \text{ m/s}}$$

sliding velocity at the disengagement $= (\omega_1 + \omega_2)\, PL$

$$= (104.72 + 62.83)\, 34.32$$

$$= 5750.32 \text{ mm/s}$$

$$= 5.75 \text{ m/s}$$

$$\boxed{\text{sliding velocity at the disengagement} = 5.75 \text{ m/s}}$$

- contact ratio $= \dfrac{\text{Length of arc of contact}}{\text{circular pitch}}$

Length of arc of contact $= \dfrac{\text{Length of path of contact (KP+PL)}}{\cos\phi}$

$$= \frac{KP + PL}{\cos\phi} = \frac{20.52 + 34.32}{\cos 20°}$$

$$= 58.36 \text{ mm}$$

$$\boxed{\text{Length of arc of contact} = 58.36 \text{ mm}}$$

circular pitch $= \pi m = \pi \times 4 = 12.57$ mm

$$\boxed{\text{circular pitch} = 12.57 \text{ mm}}$$

$$\text{contact ratio} = \frac{\text{Length of arc of contact}}{\text{circular pitch}}$$

$$= \frac{58.36}{12.57} = 4.64 \text{ say } 5$$

$$\boxed{\text{Contact ratio} = 5}$$

Result:-

1). sliding velocity at the engagement = 3.44 m/s

2). sliding velocity at the disengagement = 5.75 m/s

3). contact ratio = 5

—x—

6). Two mating gears have 20 and 40 involute teeth of module 10mm and 20° pressure angle. The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half of the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.

Given data:-

$t = 20$

$T = 40$

module $m = 10mm$

pressure angle $\phi = 20°$

The path of approach and the path of recess = half the maximum possible length.

To find:-

1). Addendum for larger Gear

2). Addendum for smaller Gear

3). length of path of contact

4). Arc of contact

5). contact ratio.

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...T. $KP = \frac{r \sin\phi}{2}$, $PL = \frac{R\sin\phi}{2}$

Addendum for larger gear = $R_A - R$ $\quad (\because R_A = R + \text{addendum})$

$$R = \frac{m \cdot T}{2} = \frac{10 \times 40}{2} = 200\,mm$$

Addendum for smaller gear = $r_A - r$ $\quad [\because r_A = r + \text{addendum}]$

$$r = \frac{m \cdot t}{2} = \frac{10 \times 20}{2} = 100\,mm$$

$$\because KP = \frac{r \sin\phi}{2}$$

$$\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi = \frac{r\sin\phi}{2}$$

$$\sqrt{R_A^2 - 200^2\cos^2 20} - 200\sin 20^\circ = \frac{100\sin 20^\circ}{2}$$

$$\cancel{\sqrt{R_A^2} - \sqrt{200^2\cos^2 20} - 200\sin 20^\circ = \frac{100\sin 20^\circ}{2}}$$

$$\cancel{\sqrt{R_A^2} - \sqrt{35320.89} - 68.40 = 17.10}$$

$$\cancel{\sqrt{R_A^2} - 187.93 - 68.40 = 17.10}$$

$$\cancel{\sqrt{R_A^2} = 17.10 + 187.93 + 68.40}$$

$$\cancel{\sqrt{R_A^2} = 273.43}$$

~~squaring on both side~~

$$\cancel{\left(\sqrt{R_A^2}\right)^2 = (273.43)^2}$$

$$\cancel{R_A^2 = 74763.96}$$

$$\cancel{R_A =}$$

$$\sqrt{R_A^2 - 200^2\cos^2 20^\circ} = \frac{100\sin 20^\circ}{2} + 200\sin 20^\circ$$

$$\sqrt{R_A^2 - 35320.89} = 68.40 + 17.10$$

$$\sqrt{R_A^2 - 35320.89} = 85.5$$

squaring on both side

$$\left(\sqrt{R_A^2 - 35320.89}\right)^2 = (85.5)^2$$

$$R_A^2 - 35320.89 = 7310.25$$

$$R_A^2 = 7310.25 + 35320.89$$

$$R_A^2 = 42631.14$$

$$R_A = \sqrt{42631.14} = 206.47\,mm$$

$$R_A = 206.47\,mm$$

$\therefore$ Addendum height for larger gear wheel $= R_A - R$

$= 206.47 - 200$

$= 6.47\,mm$

similarly

$$PL = \frac{R\sin\phi}{2}$$

$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

$$\sqrt{r_A^2 - 100^2\cos^2 20^\circ} - 100\sin 20^\circ = \frac{200\sin 20^\circ}{2}$$

$$\sqrt{r_A^2 - 100^2\cos^2 20^\circ} = \frac{200\sin 20^\circ}{2} + 100\sin 20^\circ$$

$$\sqrt{r_A^2 - 8830.22} = 34.20 + 34.20$$

$$\sqrt{r_A^2 - 8830.22} = 68.40$$

squaring on both side

$$\left(\sqrt{r_A^2 - 8830.22}\right)^2 = (68.40)^2$$

$$r_A^2 - 8830.22 = 4678.56$$

$$r_A^2 = 4678.56 + 8830.22$$

$$r_A^2 = 13508.78$$

$$r_A = \sqrt{13508.78} = 116.23\,mm$$

$$r_A = 116.23\,mm$$

$\Rightarrow$ Addendum ...

...eight for smaller gear $= r_A - r$

$= 116.23 - 100 = 16.23\,mm$

$= 16.23\,mm$

Length of Path of contact $KL = KP + PL$

$$KP + PL = \frac{r \sin\phi}{2} + \frac{R \sin\phi}{2}$$

$$= \frac{100 \sin 20^\circ}{2} + \frac{200 \sin 20^\circ}{2}$$

$$= 34.20 + 17.10 = 51.3\,mm$$

$$\boxed{KL = 51.3\,mm}$$

$$\text{Length of arc of contact} = \frac{\text{Length of Path of contact}}{\cos\phi}$$

$$= \frac{51.3}{\cos 20^\circ} = 54.59\,mm$$

$$\text{contact ratio} = \frac{\text{length of arc of contact}}{\text{circular pitch } (\pi m)}$$

$$= \frac{54.59}{\pi \times 10} = 1.74 \text{ say } 2$$

Result:-

1). Addendum for larger wear $= 6.47\,mm$
2). Addendum for smaller wear $= 16.23\,mm$
3). Length of Path of contact $= 51.3\,mm$
4). Length of ~~Path~~ arc of contact $= 54.59\,mm$
5). contact ratio $= 2$

(7). A pair of involute spur gears with 16° pressure pitch of module 6 mm is in mesh. The number in pinion is 16 its rotational speed is 240 rpm gear ratio is 1.75. In order to avoid interference, determine 1). addenda on pinion and wheel 2). length of path of contact 3). maximum velocity of sliding on either side of pitch point.

Given data:-

pressure angle $\phi = 16°$

module $m = 6mm$

teeth on pinion $t = 16$

speed $N_1 = 240$ rpm

Gear ratio $G = 1.75$

To find:-

1). addenda on pinion & wheel

2). Length of path of contact

3). maximum velocity of sliding on either side of pitch point

solution:-

$$\text{Addenda on pinion } A_p = \frac{m \cdot t}{2}\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + 1.75(1.75+2)\sin^2 16°} - 1\right]$$

$$= 48\left[\sqrt{1.498} - 1\right] = 48 \times 0.224$$

$$\boxed{A_p = 10.76mm}$$

$$\text{Addenda on wheel } (A_w) = \frac{m \cdot T}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]$$

WKT $G = T/t \Rightarrow T = G \times t = T = 1.75 \times 16$

$T = 28$

(9)

$$A_W = \frac{6\times 28}{2}\left[\sqrt{1+\frac{1}{1.75}\left(\frac{1}{1.75}+2\right)\sin^2 16} - 1\right]$$

$$= 84\left[\sqrt{1+0.57(0.57+2)(\sin 16^\circ)^2} - 1\right]$$

$$= 84\left[\sqrt{1.111} - 1\right] = 84(1.054-1) = 4.55\text{mm}$$

$$\boxed{A_W = 4.55\text{mm}}$$

Length of path of contact $KL = KP + PL$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$R_A = R + \text{addendum}$

$$R = \frac{mT}{2} = \frac{6\times 28}{2} = 84\text{mm}$$

$R_A = R + \text{addendum} = 84 + 4.55 = 88.55\text{mm}$

$R_A = 88.55\text{mm}$

$r_A = r + \text{addendum}$

$$r = \frac{mt}{2} = \frac{6\times 16}{2} = 48\text{ mm}$$

$r_A = r + \text{addendum} = 48 + 10.76 = 58.76\text{mm}$

$r_A = 58.76\text{mm}$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{88.55^2 - 84^2\cos^2 16} - 84\sin 16^\circ$$

$$= \sqrt{7841.10 - 6519.91} - 23.15$$

$$= 36.34 - 23.15 = 13.20\text{mm}$$

$$\boxed{KP = 13.20\text{mm}}$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$
$$= \sqrt{58.76^2 - 48^2\cos^2 16^\circ} - 48\sin 16^\circ$$
$$= \sqrt{3452.73 - 2128.95} - 13.23$$
$$= 36.38 - 13.23 = 23.15\,mm$$

$\boxed{PL = 23.15\,mm}$, $KL = KP + PL = 13.20 + 23.15 = 36.35\,mm$

length of path of contact = 36.35mm

maximum velocity of sliding on either side of pitch point (i.e velocity of sliding at the engagement & at the disengagement)

velocity of sliding at the engagement $= (\omega_1 + \omega_2)\,KP$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2 \times \pi \times 240}{60} = 25.13\ rad/sec$$

WKT $\frac{\omega_1}{\omega_2} = T/t \Rightarrow \cancel{\omega_2 = \omega_1 \times T/t}$

$\cancel{\omega_2 = 25.13 \times 28/16}$

$$\frac{\omega_1}{\omega_2} = UT \Rightarrow \omega_2 = \frac{\omega_1}{UT} = \frac{25.13}{1.75}$$
$$\omega_2 = 14.36\ rad/sec$$

$$\therefore (\omega_1 + \omega_2)\,KP = (25.13 + 14.36)\,13.20 = 521.27\,mm/s$$

velocity of sliding at the engagement = 521.27 mm/s

velocity of sliding at the disengagement $= (\omega_1 + \omega_2)\,PL$

$$= (25.13 + 14.36)\,23.15 = 914.19\ mm/s$$

Result:-

1). Addenda on pinion = 10.76mm

2). Addenda on wheel = 4.55mm

3). Length of path of Contact = 36.35mm

4). velocity of sliding at the engagement = 521.27 mm/s

5). velocity of sliding at the disengagement = 914.19 mm/s

...te gears of 20° pressure angle in mesh. (10)
...umber of teeth on pinion is 20 and the gear
...io is 2. If the pitch expressed in module
is 5 mm, and the pitch line speed is 1.2 m/s;
determine the angle turned through by pinion, when
one pair of teeth is in mesh. Also calculate the
maximum velocity of sliding. Take addendum
as one module.

Given data:-

Pressure angle $\phi = 20°$

number of teeth on pinion $t = 20$

Gear ratio $G = 2$

module $m = 5\,mm$

velocity $v = 1.2\,m/s = 1200\,mm/s$

addendum $= 1$ module $= 5\,mm$

To find:-

1). Angle turned through by Pinion

2). maximum velocity of sliding

Solution:-

$$\text{Angle turned through by pinion} = \frac{\text{length of arc of contact} \times 360°}{\text{circumference of pinion}}$$

$$\text{maximum velocity of sliding} = (\omega_1 + \omega_2)\,KP$$

$$\text{Length of arc of contact} = \frac{\text{length of path of contact (KL)}}{\cos\phi}$$

$$KL = KP + PL$$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

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Radius of addendum circle of wheel R_A =

Radius of pitch circle of wheel $R = \frac{mT}{2}$

WKT $m = T/t \Rightarrow T = m \times t$

$2 = T/20 \Rightarrow T = 40$

$\therefore R = \frac{5 \times 40}{2} = 100\,mm$

$R_A = 100 + 5 = 105\,mm$

Radius of addendum circle of Pinion $r_A = r$ + addendum

Radius of pitch circle of Pinion $r = \frac{mt}{2}$

$r = \frac{5 \times 20}{2} = 50\,mm$

$r_A = 50 + 5 = 55\,mm$

$\therefore KP = \sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi$

$= \sqrt{105^2 - 100^2 \cos^2 20^\circ} - 100 \sin 20^\circ$

$= \sqrt{11025 - 8830.22} - 34.20$

$= 46.85 - 34.20 = 12.65\,mm$

$KP = 12.65\,mm$

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{55^2 - 50^2 \cos^2 20^\circ} - 50 \sin 20^\circ$

$= \sqrt{3025 - 2207.56} - 17.10$

$= 28.59 - 17.10 = 11.49\,mm$

$PL = 11.49\,mm$

$KL = KP + PL = 12.65 + 11.49 = 24.14\,mm$

$KL = 24.14\,mm$

arc of contact = $\frac{\text{length of Path of contact}}{\cos\phi}$

$= \frac{24.14}{\cos 20^\circ} = 25.68\,mm$

$\therefore$ Angle turned through by pinion = $\frac{\text{length of arc of contact} \times 360}{\text{circumference of pinion } (2\pi r)}$

$= \frac{25.68 \times 360^\circ}{2 \times \pi \times 50}$

$= 29.43^\circ$

maximum velocity of sliding $= (\omega_1 + \omega_2)\, KP$

WKT ~~$\omega_r = V \times r$~~

Linear velocity = Angular velocity x length

$V = \omega . r$

so $\omega = V/r \Rightarrow V = \omega_1 . r = \omega_2 . R$

$\therefore \omega_1 = V/r, \quad \omega_2 = V/R$

$\omega_1 = \frac{1200}{50} = 24\ rad/s$

$\omega_2 = \frac{1200}{100} = 12\ rad/s$

$\therefore$ velocity of sliding $= (\omega_1 + \omega_2)\, KP$

$= (24 + 12)\, 12.65$

$= 455.4\, mm/s$

Result:-

1). Angle turned through by pinion $= 29.43^\circ$

2). maximum velocity of sliding $= 455.4\ mm/s$

(9). A pinion having 20 involute teeth of module rotates at 200 rpm and transmits 1.5 kW to wheel having 50 teeth. The addendum of both the is 1/4 of the circular pitch. The angle of obliquity 20°. Find i). length of path of approach ii). the length of arc of approach iii). the normal force between the teeth at an instant where there is only pair of teeth in contact.

Given data:-

$t = 20$

module $m = 6\,mm$

$N_1 = 200\,rpm$

$T = 50$

Angle of obliquity (or) pressure angle $\phi = 20°$

Power $P = 1.5\,kW = 1.5 \times 10^3$

addendum for wheel & pinion = 1/4 of circular pitch

$= \frac{1}{4} \times \pi m = \frac{1}{4} \times \pi \times 6 = 4.71\,mm$

To find:-

i). length of path of approach (KP)

ii). Length of arc of approach

iii). The normal force between the teeth

solution

length of path of approach (KP) $= \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

length of arc of approach $= \dfrac{\text{length of path of approach}}{\cos\phi}$

The normal force between the teeth:-

WKT Power = normal force (F) × pitch line velocity (V)

$P = F \times V$

$F = P/V$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$R_A = R + \text{addendum}$

$$R = \frac{m \cdot T}{2} = \frac{6 \times 50}{2} = 150\,mm$$

$R_A = R + \text{addendum}$

$= 150 + 4.71 = 154.71\,mm$

$$KP = \sqrt{154.71^2 - 150^2\cos^2 20} - \cancel{100}\,150 \sin 20$$

$$= \sqrt{23935.18 - 22500\,(\cos 20^\circ)^2} - \cancel{100}\,150 \sin 20^\circ$$

$$= \sqrt{4067.18} - \cancel{34.20}\ 51.30$$

$KP = 12.47\,mm$

Length of arc of approach $= \dfrac{KP}{\cos\phi} = \dfrac{12.47}{\cos 20^\circ} = 13.27\,mm$

Force $F = \dfrac{P}{V}$

$V = \dfrac{\pi d_1 N_1}{60}$, $\quad r = \dfrac{m \cdot t}{2} = \dfrac{6(20)}{2} = 60\,mm$

$d = 2r = 2 \times 60 = 120\,mm$

$$= \frac{\pi \times 120 \times 10^{-3} \times 200}{60}$$

$V = 1.26\,m/s$

$$F = \frac{1.5 \times 10^3}{1.256} = 1190.48\,N$$

Result:-

1). Length of path of approach (KP) = 12.47 mm

2). Length of arc of the approach = 13.27 mm

3). The normal force between the teeth = 1190.48 N

— x —

(10). Two mating involute spur gears of 20° have a gear ratio of 2. The number of teeth on pinion of 20 and its speed is 250 rpm. The pitch of the teeth is 12 mm. If the addendum of each wheel is such that the path of approach and the path of recess on each side are half the maximum possible length each, find i). the addendum for pinion and gear wheel ii). the length of arc of contact iii). the maximum velocity of sliding during approach and recess. Assume pinion to be driver.

Given data:-

Pressure angle $\phi = 20^\circ$

Gear ratio $G = 2$

$t = 20$

$N_1 = 250 \text{ rpm}$

module $m = 12 \text{ mm}$

To find:-

i). Addendum for pinion and gear wheel

ii). Length of arc of contact

iii). The maximum velocity of sliding during approach and recess.

Solution:

Addendum for pinion

$$\sqrt{r_A^2 - r^2 \cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

$$r_A = r + \text{addendum}$$

Addendum $A_P = r_A - r$

$$r = \frac{m \cdot t}{2} = \frac{12 \times 20}{2} = 120 \text{ mm}$$

$$R = \frac{m \cdot T}{2}$$

WKT $G = T/t \Rightarrow T = G \times t = 2 \times 20 = 40$

$$R = \frac{12 \times 40}{2} = 240 \text{ mm}$$

$$\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

$$\sqrt{r_A^2 - 120^2\cos^2 20^\circ} - 120\sin 20^\circ = \frac{240\sin 20}{2}$$

$$\sqrt{r_A^2 - 12715.52} = \frac{240\sin 20^\circ}{2} + 120\sin 20^\circ$$

$$\sqrt{r_A^2 - 12715.52} = 41.04 + 41.04$$

squaring on both sides

$$\left(\sqrt{r_A^2 - 12715.52}\right)^2 = (82.08)^2$$

$$r_A^2 - 12715.52 = \cancel{63}\ 6737.13$$

$$r_A^2 = 6737.13 + 12715.52$$

$$r_A = \sqrt{19452.65}$$

$$r_A = 139.47$$

$\therefore$ Addendum for pinion $= r_A - r = 139.47 - 120 = 19.47\,mm$

Addendum for wheel $= R_A - R$

$$\sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi = \frac{r\sin\phi}{2}$$

$$\sqrt{R_A^2 - 240^2\cos^2 20^\circ} - 240\sin 20^\circ = \frac{120\sin 20^\circ}{2}$$

$$\sqrt{R_A^2 - 50862.08} = \frac{120\sin 20^\circ}{2} + 240\sin 20^\circ$$

$$\sqrt{R_A^2 - 50862.08} = 20.52 + 82.08$$

squaring on both side

$$\left(\sqrt{R_A^2 - 50862.08}\right)^2 = (102.60)^2$$

$$R_A^2 - 50862.08 = 10527.75$$

$$R_A^2 = 10527.75 + 50862.08$$

$$R_A^2 = 61389.83$$

$R_A = \sqrt{61389.83}$

$R_A = 247.77\,mm$

Addendum for wheel $= R_A - R = 247.77 - 240 = 7.77\,mm$

Length of arc of contact $= \dfrac{\text{length of path of contact (KL)}}{\cos\phi}$

$KL = KP + PL$

$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{247.77^2 - 240^2\cos^2 20^\circ} - 240\sin 20^\circ$

$= \sqrt{61389.97 - 50862.08} - 82.08 = \sqrt{10527.89} - 82.08$

$= 102.60 - 82.08 = 20.53\,mm$

$KP = 20.53\,mm$

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{139.47^2 - 120^2\cos^2 20^\circ} - 120\sin 20^\circ$

$= \sqrt{19451.88 - 12715.52} - 41.04 = \cancel{682}\sqrt{6736.36} - 41.04$

$= 82.07 = 41.04 = 41.04\,mm$

$PL = 41.04$

$\therefore KL = KP + PL = 20.53 + 41.04 = 61.97\,mm$

length of arc of contact $= \dfrac{KL}{\cos\phi} = \dfrac{61.97}{\cos 20^\circ} = 65.95\,mm$

velocity of sliding during approach $= (\omega_1 + \omega_2)\,KP$

$\omega_1 = \dfrac{2\pi N_1}{60} = \dfrac{2 \times \pi \times 250}{60} = 26.18\,rad/s$

WKT $\quad G = T/t = \dfrac{\omega_1}{\omega_2} \Rightarrow G = \dfrac{\omega_1}{\omega_2} \Rightarrow \omega_2 = \dfrac{\omega_1}{2}$

$\omega_2 = \dfrac{26.18}{2} = 13.09\,rad/sec$

...sliding during approach $= (\omega_1+\omega_2)\, KP$

$= (26.18 + 13.09) \times 20.53$

$= 806.21$ mm/s

velocity of sliding during recess $= (\omega_1+\omega_2)\, PL$

$= (26.18 + 13.09) \times 41.04$

$= 1611.64$ mm/s

Result:-

1). Addendum for Pinion $= 19.47$ mm

2). Addendum for wheel $= 7.77$ mm

3). Length of arc of contact $= 65.95$ mm

4). The maximum velocity of sliding during approach $= 806.21$ mm/s

5). The maximum velocity of sliding during recess $= 1611.64$ mm/s

—x—

11. Two $20°$ pressure angle gears have a module of 4 mm. The number of teeth on pinion is 24 and it rotates at 600 rpm. Number of teeth on gear is 40. Addendum equals module for both pinion and gear. Determine the velocity of sliding at the final point of contact.

Given data:-

pressure angle $\phi = 20°$

module $m = 4$ mm

$t = 24$

$N_1 = 600$ rpm

$T = 40$

Addendum = 1 module = 4 mm

To find:-

velocity of sliding at the final point of contact

Solution:-

Velocity of sliding at the final point of contact = $(\omega_1$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\times\pi\times 600}{60} = 62.83 \text{ rad/sec}$$

$$\frac{T}{t} = \frac{\omega_1}{\omega_2} \Rightarrow \omega_2 = T/t$$

$$\frac{40}{24} = \frac{62.83}{\omega_2}$$

$$1.67 = \frac{62.83}{\omega_2} \Rightarrow \omega_2 = \frac{62.83}{1.67} = 37.62 \text{ rad/sec}$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$r_A = r +$ addendum

$$r = \frac{m \cdot t}{2} = \frac{4\times 24}{2} = 48 \text{ mm}$$

$$r_A = 48 + 4 = 52 \text{ mm}$$

$$\therefore PL = \sqrt{52^2 - 48^2\cos^2 20^\circ} - 48\sin 20^\circ$$

$$= \sqrt{2704 - 2034.48} - 16.42 = \sqrt{669.52} - 16.42$$

$$= 25.88 - 16.42 = 9.46 \text{ mm}$$

$\therefore$ velocity of sliding $= (\omega_1 + \omega_2)\,PL$

$$= (62.83 + 37.62)\,9.46$$

$$= 950.26 \text{ mm/s}$$

Result:-

Velocity of sliding at the final point of contact $= 950.26$ mm/s

—x—

of spur wheels with involute teeth is to give a ratio of 3 to 1. The arc of approach is not to be less than the circular pitch and the smaller wheel is the driver. The pressure angle is 20°. What is the least number of teeth that can be used on each wheel? What is the addendum of wheel in terms of circular pitch?

Given data:

Gear ratio $G = 3$

Pressure angle $\phi = 20°$

The arc of approach is no to be less than the circular pitch.

TO find:-

1). Least number of teeth on each wheel

2). Addendum of the gear wheel

solution

$$\text{The arc of approach} = \frac{\text{length of Path of approach}}{\cos\phi}$$

$$= \frac{r \sin\phi}{\cos\phi} = r\tan\phi$$

$$\text{circular Pitch } p_c = \pi m = \pi D/T = \frac{\pi 2r}{T}$$

$$p_c = \frac{2\pi r}{T}$$

Given that the arc of approach is not to be less than the circular pitch.

$$\therefore \quad r\tan\phi = \frac{2\pi r}{t}$$

$$t = \frac{2\pi \cancel{r}}{\cancel{r}\tan\phi} = \frac{2\pi}{\tan\phi} = \frac{2\times\pi}{\tan 20°}$$

$$t = 17.26 \text{ say } 18$$

$$t = 18$$

WKT $u = \frac{T}{t} \Rightarrow 3 = \frac{T}{18}$

$3 = \frac{\cancel{18}}{t} \Rightarrow t = 18 \times 3 = 54$

$\cancel{T} = 54$

$T = 54$

Addendum of the wheel $A_w = \frac{m \cdot T}{2}\left[\sqrt{1 + u\,(u+2)\sin^2\phi} - 1\right]$

$A_w = \frac{m \times 54}{2}\left[\sqrt{1 + 3(3+2)\sin^2 20^\circ} - 1\right]$

$= 27m\left[\sqrt{2.75} - 1\right]$

$= 27m(0.659)$

$A_w = \frac{m \cdot T}{2}\left[\sqrt{1 + \frac{1}{u}\left(\frac{1}{u} + 2\right)\sin^2\phi} - 1\right]$

$= \frac{m \cdot 54}{2}\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20^\circ} - 1\right]$

$= 27m\left(\sqrt{1.089} - 1\right) = 1.175m$

$A_w = 1.175m$

$\cancel{WKT}$ $P_c = \pi m$

$m = \frac{P_c}{\pi}$

$A_w = 1.175 \times P_c/\pi = 0.374\,P_c$

Result:-

1). Least number of teeth on wheel = 54

2). Least number of teeth on pinion = 18

3). Addendum of the wheel (A_w) = $0.374\,P_c$

—x—

... the length of arc of contact and maximum sliding velocity between mating gear teeth if module pitch = 4·25 mm, Addendum = 1 module, pressure angle = 20°, rpm of pinion = 150, No. of teeth on gears 24 and 33.

Given data:-

module $m = 4{\cdot}25$ mm

Addendum = 1 module = 4·25 mm

Pressure angle $\phi = 20°$

Speed of pinion $N = 150$ rpm

$t = 24$

$T = 33$

To find:-

1). Length of arc of contact

2). maximum sliding velocity

Solution:-

$$\text{Length of arc of contact} = \frac{\text{Length of path of contact (KL)}}{\cos\phi}$$

$\therefore KL = KP + PL$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

~~R_A~~ & R

Radius of addendum circle of wheel $R_A = R +$ Addendum

Pitch circle radius of wheel $R = \frac{m \cdot T}{2}$.

$$= \frac{4{\cdot}25 \times 33}{2}$$

$$= 70{\cdot}13 \text{ mm}$$

$R_A = R +$ Addendum $= 70{\cdot}13 + 4{\cdot}25$

$R_A = 74{\cdot}38$ mm

Radius of addendum circle of pinion $r_A = r +$

Pitch circle radius of pinion $r = \frac{m \cdot t}{2}$

$= \cancel{4 \times 24}$

$= \frac{4.25 \times 24}{2} = 51\,mm$

$r = 51\,mm$

$\therefore r_A = r + \text{addendum} = 51 + 4.25 = 55.25\,mm$

$r_A = 55.25\,mm$

$\therefore \cancel{PL = r_A^2 - r^2 \cos^2\phi}$

$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$

$= \sqrt{74.38^2 - 70.13^2 \cos^2 20^\circ} - 70.13\sin 20^\circ$

$= \sqrt{5532.38 - 4342.89} - 23.99$

$= \sqrt{1189.49} - 23.99 = 34.49 - 23.99$

$KP = 10.5\,mm$

$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$

$= \sqrt{55.25^2 - 51^2\cos^2 20^\circ} - 51\sin 20^\circ$

$= \sqrt{3052.56 - 2296.74} - 17.44$

$= \sqrt{755.82} - 17.44 = 27.49 - 17.44 = 10.05\,mm$

$PL = 10.05\,mm$

$\therefore KL = KP + PL = 10.5 + 10.05 = 20.55\,mm$

Length of arc of contact $= \frac{\text{Length of Path of contact}}{\cos\phi}$

$= \frac{20.55}{\cos 20^\circ} = 21.87\,mm$

velocity of sliding $= (\omega_1 + \omega_2) KP$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2 \times \pi \times 150}{60} = 7.85 \text{ rad/sec}$$

$$\frac{\omega_1}{\omega_2} = \frac{T}{t}$$

$$\frac{7.85}{\omega_2} = \frac{33}{24} \Rightarrow \omega_2 = \frac{\cancel{33}}{\cancel{24}} \times \frac{7.85}{1.38}$$

$$\omega_2 = \cancel{10.79 \text{ rad/s}}\ 5.71 \text{ rad/s}$$

$\therefore$ maximum velocity of sliding $= (\omega_1 + \omega_2) KP$

$$= \left(7.85 + \cancel{10.79}\ 5.71\right) \times 10.5$$

$$= 142.38 \text{ mm/s}$$

Result:-

1). Length of arc of contact $= 21.87$ mm

2). maximum velocity of sliding $= 142.38$ mm/s

①

...e angle (or) angle of obliquity :-

The angle between the common normal to two gear ...eth at the Point of contact and the common tangent at the pitch point.

Length of path of contact (or) contact length :-

The length of the common normal cut off by the addendum circles of the wheel and pinion.

Arc of contact :-

It is the path traced by a point on the pitch circle from the begining to the end of engagement of a given pair of teeth. It consists of

1). Arc of approach :- It is the portion of the path of contact from the begining of the engagement to the pitch point.

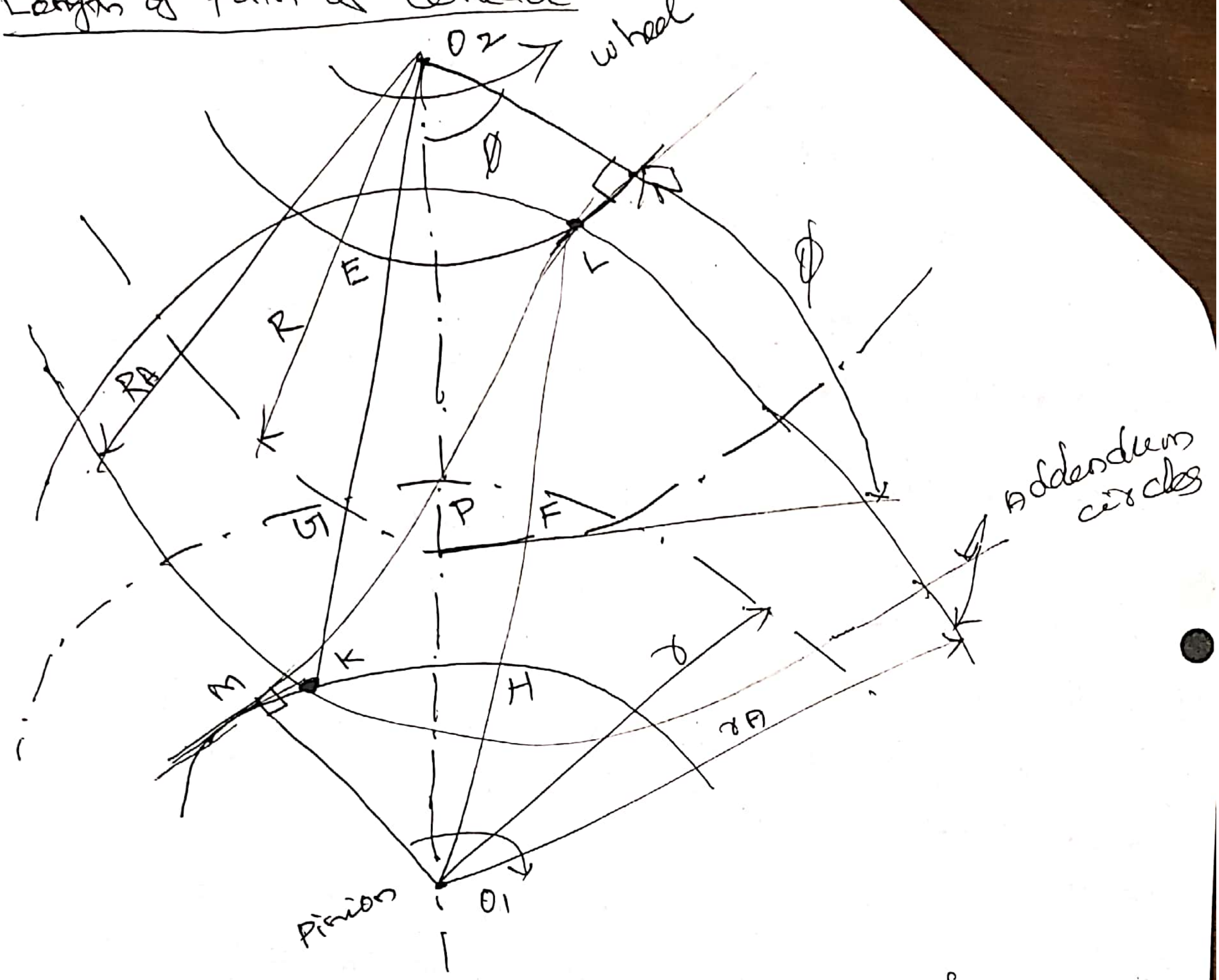
2). Arc of recess :- It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Form of Gear tooth profile

1). cycloidal tooth profile
2). Involute tooth profile

Involute	cycloidal
pressure angle constant	pressure angle varies
~~Interf~~ Interference occurs	No interference occurs
Easy to manufacture	difficult to manufacture
weaker teeth	stronger teeth
more wear & tear	less wear & tear

Length of Path of Contact



→ consider the two involute gears are in mesh.

→ ~~when~~ the pinion rotates in clockwise direction

→ The contact btn a pair of teeth begins at point K and ends at point L. Therefore the length of path of contact is KL.

→ MN is the common tangent

→ The point K is the intersection of the addendum circle of wheel and the common tangent

→ The point L is the intersection of the addendum circle of pinion and common tangent.

→ The length KP is path of approach and the length PL is path of recess.

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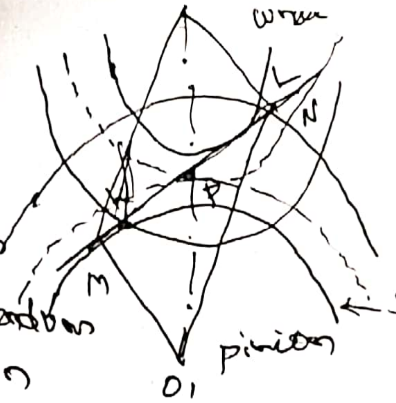

$r = O_1P$ = radius of pitch circle of pinion

$r_A = O_1L$ = radius of addendum circle of pinion

$R = O_2P$ = radius of pitch circle of wheel

$R_A = O_2K$ = radius of addendum circle of wheel

From fig; The radius of the base circle of pinion

$O_1M = O_1P\cos\phi = r\cos\phi$, similarly $O_2N = O_2P\cos\phi$
$O_2N = R\cos\phi$

Length of path of contact $KL = KP + PL$

Length of path of approach $KP = KN - PN$

Length of path of recess $PL = ML - MP$

From ΔO_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - (R\cos\phi)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

$$\therefore KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

From ΔO_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

$$\therefore PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$\boxed{\therefore KL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

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Length of arc of contact

Length of arc of contact = Length of arc of approach + length of arc of recess

$= \text{arc } GPP + \text{arc } PF$

$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi}$

$\therefore$ Length of arc of contact $= \frac{\text{Length of Path of Contact}}{\cos\phi}$

Contact ratio (or) Number of Pair of teeth in contact

contact ration $= \frac{\text{length of arc of contact}}{\text{circular Pitch } (P_c)}$

circular pitch $P_c = \frac{\pi D}{T} = \pi m$

m = module pitch (D/T)

D - dia of pitch circle, T - Number of teeth on wheel

Pitch circle radius of gear $R = \frac{m \cdot T_G}{2}$

Pitch circle radius of pinion $r = \frac{m \cdot t_p}{2}$

where m - module

T_G - No of teeth on gear wheel

t_p - No. of teeth on the pinion

Addendum radius of gear wheel $R_A = R +$ addendum

addendum radius of pinion $r_A = r +$ addendum

Angle turned through by pinion $= \frac{\text{length of arc of contact}}{\text{circumference of pinion}} \times 360°$

Gear ratio $G = T/t$

T - No. of teeth on wheel

t - No. of teeth on pinion

The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel.

(or)

The phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference.

methods to avoid interference

- The height of the teeth may be reduced
- The pressure angle may be increased
- The radial flank of the pinion may be cut back.
- The face of the gear tooth may be relieved.

minimum number of teeth on the pinion in order to avoid interference

Let
- T - Number of teeth on gear wheel
- t - number of teeth on pinion
- m - module of the teeth
- r - pitch circle radius of pinion $\left(r = \frac{mt}{2}\right)$
- R - pitch circle radius of wheel $\left(R = \frac{mT}{2}\right)$
- G - Gear ratio $\left(T/t = R/r\right)$
- ϕ - pressure angle

$$t = \frac{2A_p}{\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$$

A_p - addendum of the pinion.

$$A_p = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

minimum number of the teeth on the wheel in order to avoid interference

$$T = \frac{2A_w}{\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1} = \frac{2A_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

A_w - addendum of gear wheel

$$A_w = \frac{T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

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m Unit - V (1)

Friction

Friction — When two surfaces are in contact with each other, and one surface tends to move with respect to other, a tangential force will be developed at the contact surface, in the opposite direction of motion.

When two body surface move over another, a resisting force exist in between the two body surfaces to resist the relative motion.

Types of friction

1). Dry or columb or solid friction

The friction that exist between two unlubricated surfaces.

1). Sliding friction → The friction that exist when one surface slides over another surface

2). Rolling friction → The friction that exist when one surface rolls over another surface.

2). Skin (or) Greasy friction

The friction that exist between a minute thin layer of lubricated surfaces.

3). Film of fluid (or) viscous friction

The friction experienced between ~~two~~ rubbing surfaces when the surface have a thick layer of lubricant.

Static friction :-

The friction experienced by a body when at rest

Dynamic friction

The friction experienced by a body when in motion

Laws of dry friction

1). The frictional force is directly proportional to the normal reaction between the surfaces.

2). The frictional force opposes the motion

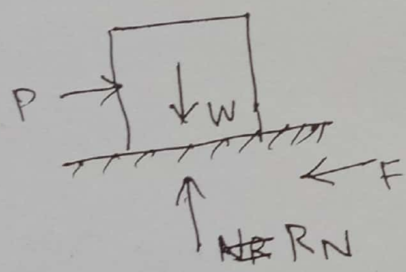
3). The frictional force depends upon the nature of the surfaces in contact.

4). The frictional force is independent of the area and the shape of the contacting surfaces.

5). The frictional force is independent of the velocity of sliding of one body relative to the other body.

Limiting friction (F)

The maximum resistance offered by the body is called limiting friction.

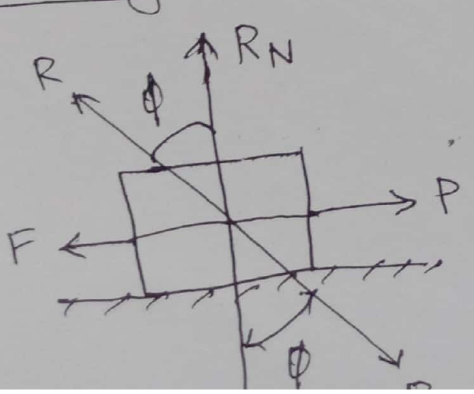


Co-efficient of friction (μ)

The ratio of limiting friction to normal reaction

$$\mu = \frac{\text{Limiting friction}}{\text{Normal reaction}} = \frac{F}{R_N}$$

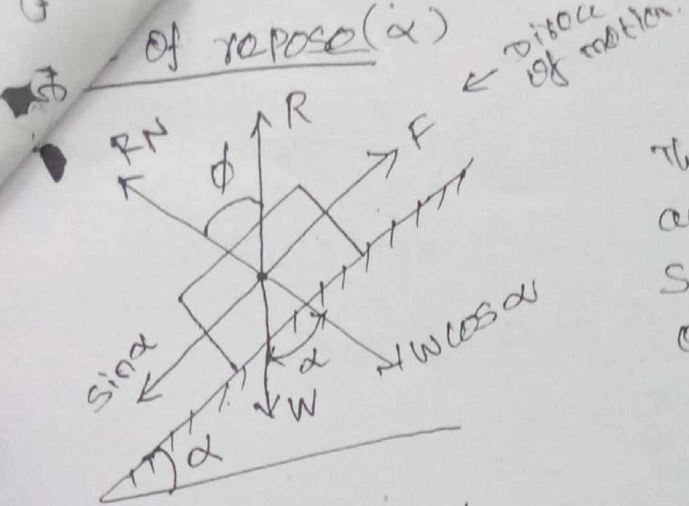
Limiting angle of friction (ϕ)



The angle at which the resultant reaction R makes with the normal reaction R_N

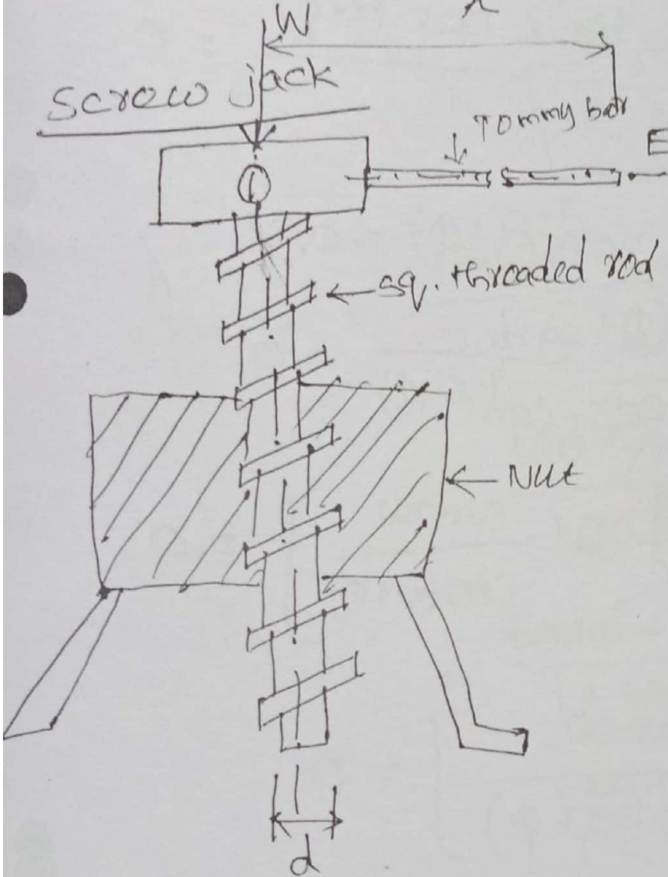
$$\boxed{\tan\phi = \mu}$$

... of repose (α)



The angle of the inclined plane at which the body tends to slide down is known as angle of repose.

Screw jack



W – Load to be lifted

P – Effort applied to lift the load

p – pitch of the screw

d – mean dia of the screw

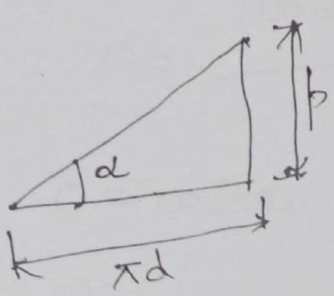
α – Helix angle

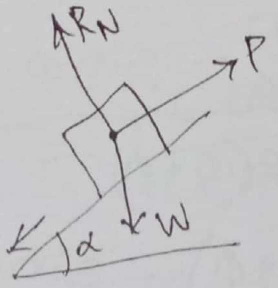
μ – co-efficient of friction

l – Horizontal distance btn centre axis of the screw and the end E of the bar

$$\tan\alpha = \frac{p}{\pi d}$$

Screw move upward





$$P = W\tan(\alpha+\phi)$$

$$M.A = \frac{W}{P} = \frac{W}{W\tan(\alpha+\phi)}$$

$$= \frac{1}{\tan(\alpha+\phi)} = \cot(\alpha+\phi)$$

Screw move downward

$$P = W\tan(\alpha-\phi) \quad P = W\tan(\phi-\alpha)$$

$$\text{mechanical efficiency} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

For determining max. efficiency

$\frac{u}{v}$ $\frac{v\,du - u\,dv}{v^2}$

$$\frac{d\eta_{UP}}{d\alpha} = 0$$

$$\eta_{UP} = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$\frac{d\eta_{UP}}{d\alpha} = \frac{\tan(\alpha+\phi)\cdot\sec^2\alpha - \tan\alpha\,\sec^2(\alpha+\phi)}{\tan^2(\alpha+\phi)} = 0$$

$$\tan(\alpha+\phi)\sec^2\alpha - \tan\alpha\sec^2(\alpha+\phi) = 0$$

$$\frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}\cdot\frac{1}{\cos^2\alpha} - \frac{\sin\alpha}{\cos\alpha}\,\frac{1}{\cos^2(\alpha+\phi)} = 0$$

$$\frac{1}{\cos\alpha\,\cos(\alpha+\phi)}\left[\frac{\sin(\alpha+\phi)}{\cos\alpha} - \frac{\sin\alpha}{\cos(\alpha+\phi)}\right] = 0$$

$$\left[\frac{\sin(\alpha+\phi)}{\cos\alpha} - \frac{\sin\alpha}{\cos(\alpha+\phi)}\right] = 0$$

$$\frac{\sin(\alpha+\phi)\cdot\cos(\alpha+\phi) - \sin\alpha\cos\alpha}{\cos\alpha\cdot\cos(\alpha+\phi)} = 0$$

$$\sin(\alpha+\phi)\cdot\cos(\alpha+\phi) - \sin\alpha\cos\alpha = 0$$

multiply by 2 on both sides

$$2\sin(\alpha+\phi)\cdot\cos(\alpha+\phi) - 2\sin\alpha\cos\alpha = 0$$

$$2\sin(\alpha+\phi)\cdot\cos(\alpha+\phi) = 2\sin\alpha\cos\alpha$$

$$\sin 2(\alpha+\phi) = \sin 2\alpha$$

$$\sin 2(\alpha+\phi) = \sin(\pi - 2\alpha) \qquad \sin\theta = \sin(\pi-\theta)$$

$$\sin 2(\alpha+\phi) = \sin(\pi - 2\alpha)$$

$$2(\alpha+\phi) = \pi - 2\alpha$$

$$2\alpha + 2\phi = \pi - 2\alpha$$

$$2\alpha + 2\alpha = \pi - 2\phi$$

$$4\alpha = \pi - 2\phi$$

$$\alpha = \frac{\pi - 2\phi}{4}$$

$$\boxed{\alpha = \pi/4 - \frac{\phi}{2}}$$

$$\eta_{max} = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan(\pi/4 - \phi/2)}{\tan(\pi/4 - \phi/2 + \phi)}$$

$$= \frac{\tan(\pi/4 - \phi/2)}{\tan(\pi/4 + \phi/2)} = \frac{\tan(45^\circ - \phi/2)}{\tan 45^\circ + \phi/2}$$

$$= \frac{\dfrac{\tan 45^\circ - \tan\phi/2}{1 + \tan 45 \cdot \tan\phi/2}}{\dfrac{\tan 45 + \tan\phi/2}{1 - \tan 45 \cdot \tan\phi/2}} = \frac{\tan 45^\circ - \tan\phi/2}{1 + \tan 45^\circ \cdot \tan\phi/2} \times \frac{1 - \tan 45^\circ \cdot \tan\phi/2}{\tan 45^\circ + \tan\phi/2}$$

$$= \frac{1 - \tan\phi/2}{1 + \tan\phi/2} \times \frac{1 - \tan\phi/2}{1 + \tan\phi/2}$$

$$= \frac{(1 - \tan\phi/2)^2}{(1 + \tan\phi/2)^2} = \frac{\left[1 - \dfrac{\sin\phi/2}{\cos\phi/2}\right]^2}{\left[1 + \dfrac{\sin\phi/2}{\cos\phi/2}\right]^2}$$

$$= \frac{\left[\dfrac{\cos\phi/2 - \sin\phi/2}{\cos\phi/2}\right]^2}{\dfrac{\cos\phi/2 + \sin\phi/2}{\cos\phi/2}}$$

$$= \frac{(\cos\phi/2 - \sin\phi/2)^2}{[\cos\phi/2 + \sin\phi/2]^2} = \frac{\cos^2\phi/2 + \sin^2\phi/2 - 2\cos\phi/2 \sin\phi/2}{\cos^2\phi/2 + \sin^2\phi/2 + 2\cos\phi/2 \sin\phi/2}$$

$$= \frac{1 - 2\cos\phi/2 \sin\phi/2}{1 + 2\cos\phi/2 \sin\phi/2} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$$\boxed{\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}}$$

$[\because 2\sin\frac{\phi}{2}\cos\frac{\phi}{2} = \sin\phi]$

Total torque required to overcome the friction $\}$ $T = T_1 + T_2$

$$T_1 = P \times d/2 = W \tan(\alpha + \phi) \times d/2$$

$$T_2 = \mu_1 W R$$

mean radius of the collar $R = \frac{R_1 + R_2}{2}$

R_1 - outside radius of the collar

R_2 - Inside radius of the collar

$$\boxed{T = P \times d/2 + \mu_1 W R}$$

If F is effort applied ~~bar~~ tangentially at the end of F of a tommy bar,

$$T = F \cdot l = W \times d/2 \times \tan(\alpha + \phi)$$

Self locking

$\eta < 50\%$ (or) $\phi > \alpha$

overhauling

~~$\eta \le 5$~~ $\eta > 50\%$. $\phi < \alpha$

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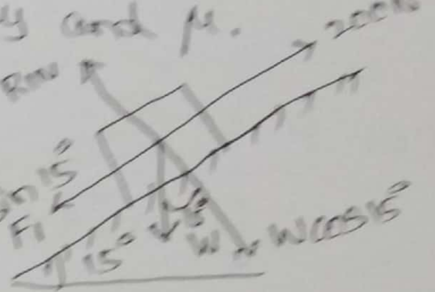
(4)

An effort of 200N is required just to move a certain body up an inclined plane of angle 15°, the force acting parallel to the plane. If the angle of inclination of the plane is made 20°, the effort required again applied parallel to the plane is found to be 230N. Find the weight of the body and μ.



Soln

In both cases, the effort is applied parallel to the inclined plane and body is just to move up. Hence the force of friction acting downwards.

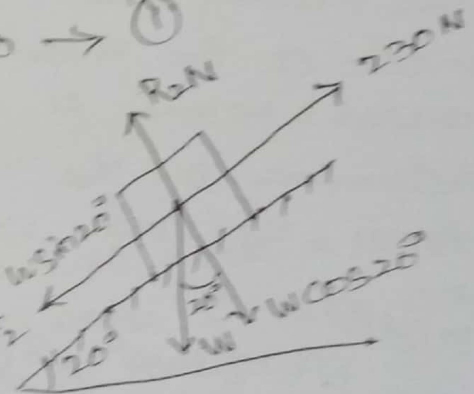
i). $W\sin 15^\circ + F_1 = 200$

$F_1 = \mu R_{1N}$

$R_{1N} = W\cos 15^\circ$

$W\sin 15^\circ + \mu W\cos 15^\circ = 200$

$W(\sin 15^\circ + \mu \cos 15^\circ) = 200 \rightarrow (1)$



ii) resolve the forces along the plane

$F_2 + W\sin 20^\circ = 230$

$F_2 = \mu R_{2N}$

Resolve the forces normal to the plane

$R_{2N} = W\cos 20^\circ$

$W\sin 20 + \mu W\cos 20^\circ = 230 \rightarrow (2)$

$W[\sin 20 + \mu \cos 20^\circ] = 230 \rightarrow (2)$

(2) ÷ (1) ⇒ $\frac{W[\sin 20 + \mu\cos 20^\circ] = 230}{W[\sin 15^\circ + \mu\cos 15^\circ] = 200}$

$\frac{0.342 + 0.939\mu}{0.258 + 0.965\mu} = 1.15$

$0.342 + 0.939\mu = 1.15[0.258 + 0.965\mu]$

$$0.342 + 0.939\mu = 0.297 + 1.1098\mu$$

$$0.342 - 0.297 = 1.1098\mu - 0.939\mu$$

$$0.045 = 0.1708\mu$$

$$\boxed{\mu = 0.263}$$

Put μ in (1)

$$W[\sin 15^\circ + \mu \cos 15^\circ] = 200$$

$$W(\sin 15^\circ + 0.26 \cos 15^\circ] = 200$$

$$W(0.258 + 0.2509) = 200$$

$$\boxed{W = 393\,N}$$

(2). The following data are related to a screw jack; pitch of the thread screw = 8mm, Dia of the screw thread = 40mm μ btn screw & Nut = 0.1, load = 20kN determine 1). The ratio of torques required to raise and lower the load, 2). The efficiency of the machine.

Given:-

$p = 8mm = 8 \times 10^{-3}\,m$

$d = 40mm = 40 \times 10^{-3}\,m$

$\mu = 0.1$

$W = 20kN = 20 \times 10^{3}\,N$

To find:-

1). Ratio of torques

2). η

solution

$$T_1 = P \times d/2 \qquad T_2 = P \times d/2$$

$$\tan\phi = \mu$$

$$T_1 = W \tan(\alpha + \phi) \times d/2 \qquad \phi = \tan^{-1}\mu$$

$$\tan\alpha = \frac{P}{\pi d} = \frac{8 \times 10^{-3}}{\pi \times 40 \times 10^{-3}} = 0.0636 \qquad \phi = 5.71^\circ$$

$$\alpha = 3.64^\circ$$

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$T_1 = W \tan(\alpha+\phi) \cdot d/2$

$= 20\times10^3 \tan(3.64+5.71) \times \frac{40\times10^{-3}}{2}$

$T_1 = 65.86$ Nm

$T_2 = W \tan(\phi-\alpha) \times d/2$

$= 20\times10^3 \tan(5.71-3.64) \times \frac{40\times10^{-3}}{2}$

$= 14.45$ Nm

$\frac{T_1}{T_2} = \frac{65.86}{14.45} = 4.55 \Rightarrow \boxed{T_1/T_2 = 4.55}$

$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan 3.64}{\tan(3.64+5.71)} = 0.3863$

$\boxed{\eta = 38.63\%}$

(3). An effort of 1500 N is required to just move a certain body up an inclined plane of angle 12°, force acting parallel to the plane. If an angle of inclination is increased to 15° then the effort required is 1720 N. Find the weight of the body & μ. (4464N, 0.13)

(4). A bolt with a square threaded screw has mean diameter of 25 mm and a pitch of 3mm. It carries an axial thrust of 10 KN on the bolt head of 25mm mean radius. If μ = 0.12 find the force required at the end of a spanner 450 mm long in tightening up the bolt.

Given:-

square thread

Mean dia $d_o = 25mm = 25\times10^{-3}\,m$

pitch $p = 3mm = 3\times10^{-3}\,m$

Load $W = 10KN = 10\times10^{3}\,N$

mean Radius $R = 25mm = 25\times10^{-3}\,m$

$\mu = 0.12$

$l = 450mm = 0.45m$

To find:- Force required at the end of spanner (F)

Solution:-

$$T = F\times l \Rightarrow F = \frac{T}{l}$$

$$T = P\times d/2 + \mu W R$$

$$T = W\tan(\alpha+\phi)\times d/2 + \mu W R$$

$$\tan\alpha = \frac{p}{\pi d} = \frac{3\times10^{-3}}{\pi\times25\times10^{-3}} = 0.038$$

$$T = 10\times10^{3}\left[\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\cdot\tan\phi}\right]\times\frac{25\times10^{-3}}{2} + 0.12\times10\times10^{3}\times25\times10^{-3}$$

$$= 10\times10^{3}\left[\frac{0.038+0.12}{1-0.038\times0.12}\right]\times\frac{25\times10^{-3}}{2} + 30$$

$$T = 49.84\,Nm$$

$$F = \frac{49.84}{0.45} = 110.77\,N$$

(5). A vertical screw with single start square thread 50mm mean diameter and 10mm pitch is raised against a load of 5500N by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 65mm. If the μ is 0.15 for the screw and 0.18 for the collar and tangential force applied by each hand to the wheel is 140N. Find the suitable diameter of the hand wheel.

2, 4, 7, 16, 25, 26, 28, 29, 43, 52, 53, 56, 59

A bolt having V-threads. The pitch of the thread is 5mm and V-angle is 55°. The mean dia of bolt is 20mm. The bolt is tightened by screwing a nut. The mean radius of the bearing surface of the nut is 25mm. The load on the bolt is 5000N. The μ for nut and bolt is 0.1. whereas for nut and bearing surface is 0.16. Determine the force required at the end of a spanner 0.6m long.

Given data:

pitch $p = 5mm = 5\times10^{-3} m$

V angle $2\beta = 55^\circ \Rightarrow \beta = 27.5^\circ$

mean dia $d = 20mm = 20\times10^{-3} m$

mean radi of the nut $R = 25mm = 25\times10^{-3} m$

Load $W = 5000N$

$\mu = 0.1$

$\mu_2 = 0.16$

length of spanner $l = 0.6m$

To find:-

Force required at the end of spanner

Solution:-

$T = F \times l$

$F = T/l$

$T = p \times d/2 + \mu_2 WR$

$\tan\alpha = \dfrac{p}{\pi d} = \dfrac{5\times10^{-3}}{\pi\times20\times10^{-3}} = 0.079$

$\mu_1 = \dfrac{\mu}{\cos\beta} = \dfrac{0.1}{\cos 27.5} = 0.113$

$= W\tan(\alpha+\varphi) \times d/2 + \mu_2 WR$

$= 5000\left[\dfrac{0.079 + 0.113}{1-0.079\times0.113}\right] \times \dfrac{20\times10^{-3}}{2} + 0.16\times5000\times25\times10^{-3}$

$T = 29.71\ Nm$

$\therefore F = 29.71\times0.6 = 49.53\ N$

Given data:-

mean dia $d = 50mm = 50 \times 10^{-3} m$

pitch $p = 10mm = 10 \times 10^{-3} m$

Load $W = 5500N$

mean dia of collar $D = 65mm = 65 \times 10^{-3} m$

$\mu = 0.15$

$\mu_1 = 0.18$

$P_1 = 140N$

To find:-

Diameter of the hand wheel (D_1)

Solution:

Torque applied to the hand wheel

T = Tangential load on wheel × Radius of wheel

$$T = 2P_1 \times \frac{D_1}{2}$$

$$T = P \times d/2 + \mu_1 W R$$

$$T = W \tan(\alpha + \phi) \times d/2 + \mu_1 W R$$

$$\tan\alpha = p/\pi d = \frac{10 \times 10^{-3}}{\pi \times 50 \times 10^{-3}} = 0.063$$

$$T = 5500 \left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}\right] \times d/2 + \mu_1 W R$$

$$= 5500 \left[\frac{0.063 + 0.15}{1 - 0.063 \times 0.15}\right] \times \frac{50 \times 10^{-3}}{2} + \left[0.18 \times 5500 \times \frac{65 \times 10^{-3}}{2}\right]$$

$$T = 61.83 \text{ Nm}$$

$$T = 2P_1 \times \frac{D_1}{2} \Rightarrow 61.83 = \cancel{2} \times 140 \times \frac{D_1}{\cancel{2}}$$

$$\boxed{D_1 = 0.441 m}$$

The efficiency of a screw jack is 55% when a load of 1500N is lifted by an effort applied at the end of a handle of length 0.5m. Determine the effort applied if the pitch of the screw thread is 10mm.

Given data:-

$\eta = 55\%$ $W = 1500N$ $l = 0.5m$ $p = 10mm$

To find:-

Effort applied (P)

Solution:-

$$\eta = \frac{\tan\alpha}{\tan(\phi+\alpha)} = \frac{p/\pi d}{P/W} = \frac{p \times W}{\pi d P}$$

$$0.55 = \frac{10 \times 10^{-3} \times 1500}{\pi \times d \cdot P}$$

$$\pi d P = \frac{10 \times 10^{-3} \times 1500}{0.55} = \cancel{27.27}\ 8.67$$

$$T = P \times d/2$$

$$T = \frac{8.67}{2} = 4.32\ Nm$$

(8). A 150mm diameter valve against a steam pressure of 2 MN/m² is acting is closed by means of square threaded screw 50mm in external dia with 6mm pitch. If the μ is 0.12 find the torque required to turn the handle.

Given:-

~~Dia of screw D = 50mm~~

Dia of valve $D = 150\,mm = 150 \times 10^{-3}\,m$

External dia of screw $d_o = 50\,mm = 50 \times 10^{-3}\,m$

pitch $p = 6\,mm = 6 \times 10^{-3}\,m$

$\mu = 0.12$

steam pressure $p = 2\,MN/m^2 = 2 \times 10^6\,N/m^2$

TO find:-

Torque required

solution:-

$$T = P \times d/2 = W \tan(\alpha+\phi) \cdot d/2$$

mean dia $d = d_o - p/2 = 50 - 6/2 = 47 \times 10^{-3}\,m$

$$\tan\alpha = p/\pi d = \frac{6 \times 10^{-3}}{\pi \times 47 \times 10^{-3}} = 0.0406$$

$$\alpha = \tan^{-1}(0.0406) = 2.33^\circ$$

$$\phi = \tan^{-1}(0.12) = 6.84$$

Load on the valve W = pressure × Area

$$= 2 \times 10^{6} \times \pi/4\,(50 \times 10^{-3})^2 = 35343\,N$$

$$T = 35343 \tan(2.33^\circ + 6.84^\circ) \times \frac{47 \times 10^{-3}}{2}$$

$$T = 134.07\,Nm$$

(9). A square threaded bolt of root dia 22.5mm and pitch 5mm is tightened by screwing nut whose mean dia of bearing surface is 50mm. If μ for nut and bolt is 0.1 and for nut and bearing surface 0.16. Find the force required at the end of spanner 500mm long when the load on the bolt is 10 kN.

$d = d_c + p/2$ $\quad F = 121.1\,N$

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(1)

A body of weight 450N is pulled up along an inclined plane with 30° inclination to the horizontal at a steady speed. If the co-efficient of friction between body and the plane is 0.25 and the force is applied parallel to the inclined plane, find the force required. Find also the workdone on the body, if the distance travelled by the body is 10m along the plane.

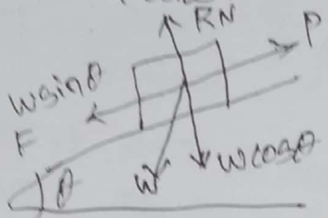
$P = F + W\sin\theta$

workdone = Force × Distance

$P \times 10m$

$P = 322\ N$

workdone = 3224 Nm



(11). A screw jack has a square thread of mean diameter 6cm and pitch 0.8cm. The μ of the screw thread is 0.09. A load of 3kN is to be lifted through 12cm. Determine the torque required and workdone in lifting the load through 12cm. Find ~~the~~ efficiency of the jack

Given:-

$d = 6cm = 6 \times 10^{-2} m$

$p = 0.8cm = 0.8 \times 10^{-2} m$

$\mu = 0.09$

$W = 3kN = 3 \times 10^{3} N$

To find:-

1). Torque required to raise the load 2). workdone in lifting the load through 12cm 3). Efficiency of the screw jack.

solution:-

$T = W \tan(\alpha + \phi) \times d/2 = 11.96\ Nm$

workdone $= 2\pi N T$ $\Rightarrow N = \frac{12}{0.8} = \frac{\text{Distance}}{\text{pitch}} = 15$

$= 1127\ Nm$

number turns required to lift the load

$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = 37\ \%$

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(12). Pitch of 50 mm dia threaded screw of a screw jack 12.5mm. μ of screw and nut is 0.10. Determine the torque required to raise the load of 25 KN rotating with the screw. Also find the torque required to lower the load and efficiency of the screw jack.

$$T_1 = W \tan(\alpha+\phi) \times d/2 \qquad T_2 = W \tan(\phi-\alpha) \cdot d/2$$

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

(13). A load of 10KN is raised by means of a screw jack, having a square threaded screw of pitch 12 mm and of mean dia 50 mm. If the force of 100N is applied at the end of a lever to raise the load, what should be the length of lever used? $\mu = 0.15$. What is the mechanical advantage obtained? State whether the screw is self locking or not.

~~Given~~:- $P_1 = 100N$

$$T = P \times d/2 \qquad T = P_1 \times l$$

$$MA = \frac{W}{P_1} = \frac{10 \times 10^3}{100} = 100$$

$\eta = 33.55\% < 50\%$ so self-locking

(14). The mean dia of the screw jack having pitch of 10mm is 50mm. A load of 20KN is lifted through a distance of 170mm. Find the work done in lifting the load and efficiency of the screw jack when

1). the load rotates with the screw

2). the load rests on the loose head which does not rotate with the screw

The external and Internal dia of the bearing surface of the loose head are 60mm and 10mm respectively. The μ for the screw as well as the bearing surface may be taken as 0.08.

Solut

Torque required to overcome friction at the screw

$$T = P \times d/2$$

N = Number of turns required to lift the load through a distance of $170mm = \frac{170}{10} = 17$

$$\text{workdone} = 2\pi NT$$

$= \dfrac{\tan\alpha}{\tan(\alpha+\phi)}$

total torque required to overcome the friction at the screw and collar

$$T = P \times d/2 + \mu_1 W R$$

work done $= 2\pi N T$

$\eta = \dfrac{T_0}{T} \Rightarrow T_0 = P_0 \times d/2 \Rightarrow T_0 = W \tan\alpha \times d/2$ (neglecting friction)

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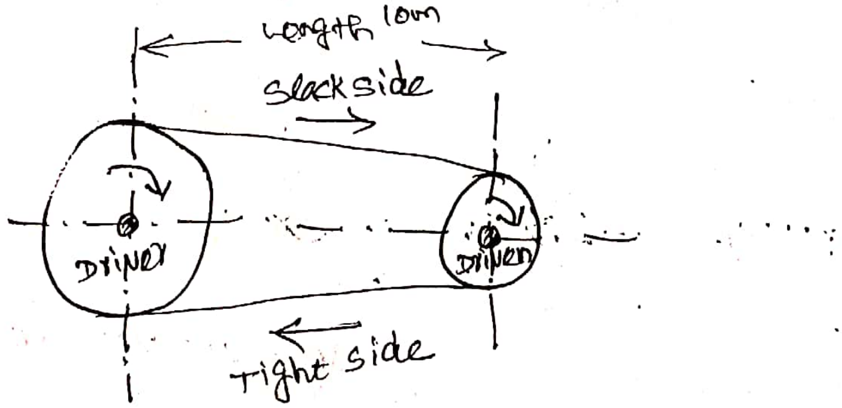
Bolt and rope drives

- The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

The amount of power transmitted depends upon the
- velocity of the belt
- Tensions in the belt
- The arc of contact between the belt and the smaller pulley
- the conditions under which the belt is used



Selection of a belt drive

The following factors are consider while selecting the belt drive

1. speed of the driving and driven shaft
2. speed reduction ratio
3. Power to be transmitted
4. centre distance between the shafts
5. Positive drive requirements
6. shafts layout
7. space available
8. service conditions

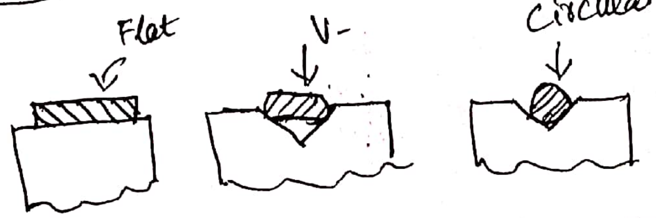
Types of Belt drives

1. Light drives – Transmits small powers at belt speeds up to 10 m/s. Uses:- Agricultural machi… and small machine tools.

2. Medium drives – Transmit medium power at belt speeds above 10 m/s but less than 22 m/s Uses:- machine tools

3. Heavy drives – Transmit large power at belt speeds above 22 m/s. Uses:- compressors and generators.

Types of belts



1. Flat belt – when the two pulleys are not more than 8m moderate power transmitted
2. V- belt – when the two pulleys very nearer to each other moderate power transmitted
3. circular belt – when the two pulleys are more than 8m, great amount of power to be transmitted

Belt material → must be strong, flexible and durable, high μ

1. Leather belts – 1.2m to 1.5m long strips, hair side and flesh side
2. Cotton or fabric belts – Folding canvass or cotton duck stitching together.
3. Rubber belt – Very flexible but quickly destroyed
4. Balata belts – acid proof & water proof.

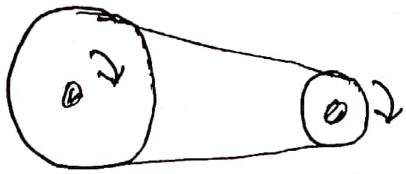
Flat belts – Leather canvas, Cotton and rubber

V-belts – Rubberised fabric and rubber

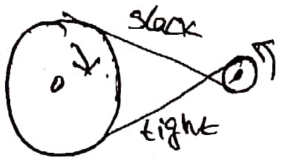
Ropes – cotton, hemp and manila

# of Flat belt drives

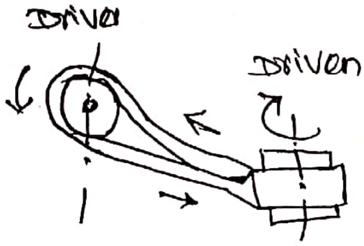
1. Open belt drive



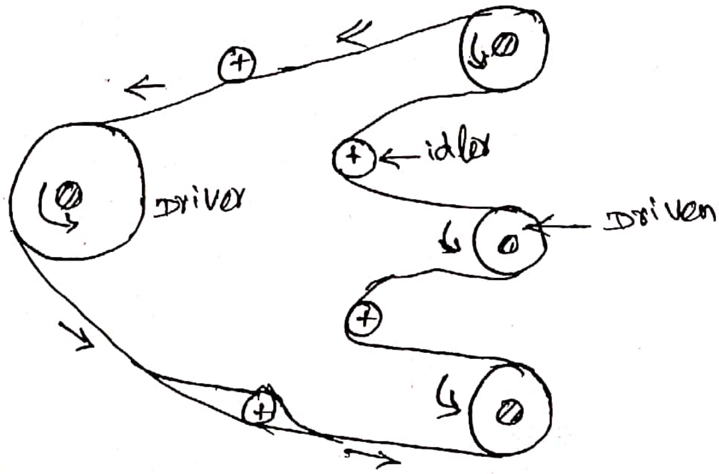
2. Twist (or) cross belt drive



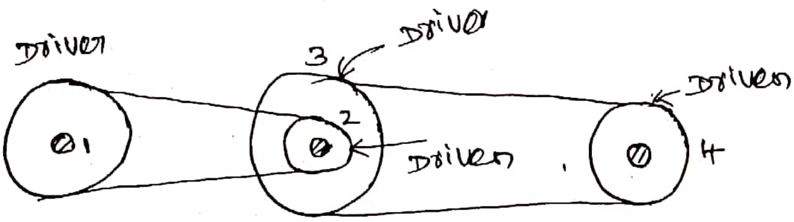
3. Quarter turn belt drive



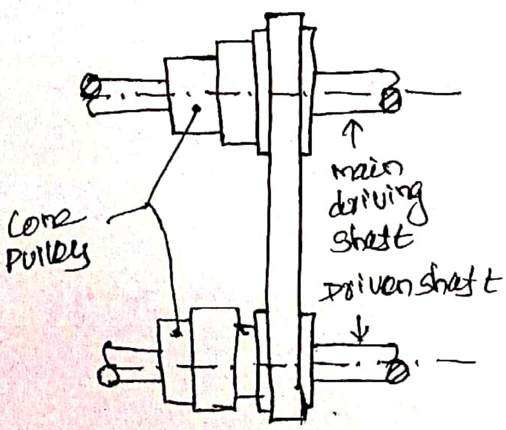
4. Belt drive with idler pulleys



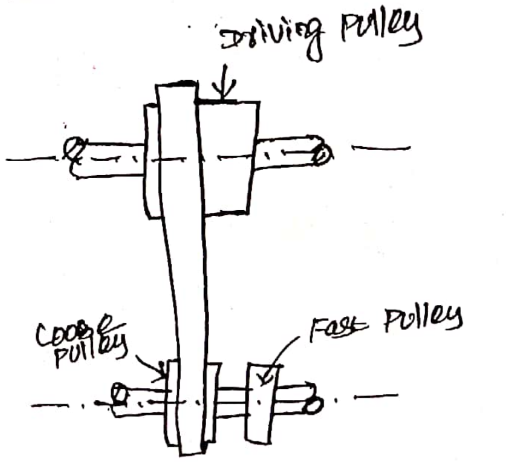
5. Compound belt drive



6. Stepped or Cone pulley drive



7. Fast and loose pulley drive



Velocity ratio of belt drive

The ratio between the velocities of driver and the driven or follower.

Let, d_1 - Diameter of the driver

d_2 - Diameter of the driven

N_1 - speed of the driver in rpm

N_2 - speed of the driven in rpm

Velocity of the belt passes over the driver

$$V_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

Velocity of the belt passes over the driven

$$V_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$

Assuming no slip between the belt and pulley

$$V_1 = V_2$$

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

$\therefore$ velocity ratio, $\boxed{\frac{d_1}{d_2} = \frac{N_2}{N_1}}$

When the thickness of belt is considered, then the velocity ratio

$$\boxed{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}}$$

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... of belt

The relative motion between the belt and pulley. The frictional grip between belt and pulley is insufficient. The presence of slip reduces the velocity ratio of the drive.

Let S_1 – % of slip between belt and driver

S_2 – % of slip between belt and follower

∴ velocity of the belt passing over the pulley driver

$$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100}$$

$$V = \frac{\pi d_1 N_1}{60}\left[1 - \frac{S_1}{100}\right]$$

$$V = v\left(1 - \frac{S_1}{100}\right)$$

Velocity of the belt passing over the follower

$$V = \frac{\pi d_2 N_2}{60} - \frac{\pi d_2 N_2}{60} \times \frac{S_2}{100}$$

$$V = v\left[1 - \frac{S_2}{100}\right]$$

$$\therefore \frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1}{100} - \frac{S_2}{100}\right)$$

$$= \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right)$$

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}\left[1 - \frac{S}{100}\right]} \qquad [\because S = S_1 + S_2]$$

If thickness of belt is considered

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)$$

Velocity ratio of compound belt drive

$$\text{Velocity ratio} = \frac{\text{speed of last driven}}{\text{speed of first driver}}$$

$$= \frac{\text{~~speed~~ product of diameters of drivers}}{\text{product of diameters of drivens}}$$

Creep of Belt

A certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and pulley surfaces. This relative motion is termed as creep.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

$$\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

E – Young's modulus of belt material

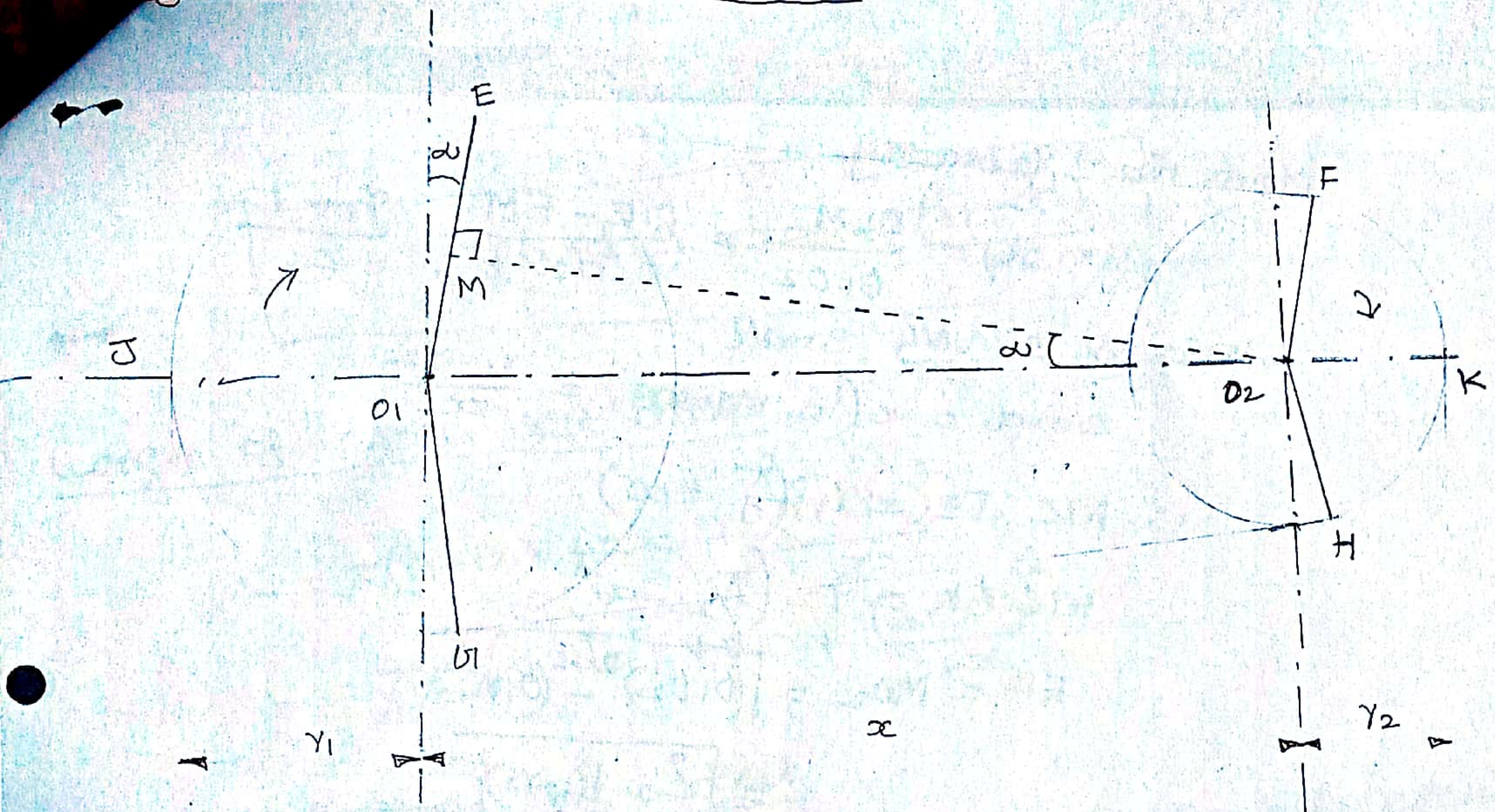
σ_1 – stress in the belt on the tight side

σ_2 – stress in the belt on the slack side

Creep

continuous deformation of metals under steady load.

Length of an open belt drive



In open belt drive both the pulleys rotate in the same direction.

Let r_1 and r_2 - Radii of the larger and smaller pulleys

x - Distance between the centres of two pulleys

L - Total length of the belt

The belt leaves the larger pulley at E and G

The belt leaves the smaller pulley at F and H

$O_2M \parallel FE$

Angle $MO_2O_1 = \alpha$ radians

Length of the belt $L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$

But Arc JE = Arc GJ

$EF = GH$

Arc FK = Arc KH

$$L = 2\left[\text{Arc } JE + EF + \text{Arc } FK\right]$$

From the geometry

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

since α is very small,

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x}$$

$$\therefore \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right)$$

$$\text{Arc } FK = r_2\left(\pi/2 - \alpha\right)$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{x^2 - (r_1 - r_2)^2}$$

$$EF = x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding the above EF by binomial theorem,

$$EF = x\left[1 - \tfrac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \dots\right] = x - \frac{(r_1 - r_2)^2}{2x}$$

$$\therefore L = 2\left[\text{Arc } JE + EF + \text{Arc } FK\right]$$

$$= 2\left[r_1(\pi/2 + \alpha) + x - \frac{(r_1 - r_2)^2}{2x} + r_2(\pi/2 - \alpha)\right]$$

$$= 2\left[r_1\,\pi/2 + r_1\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2\,\pi/2 - r_2\alpha\right]$$

$$= 2\left[\pi/2(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 \cancel{+} r_2)^2}{x}$$

Put $\alpha = \frac{r_1 - r_2}{x}$

$$L = \pi(r_1 + r_2) + 2\left(\frac{r_1 - r_2}{x}\right) \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2\frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}}$$

(or)

$$\boxed{L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}}$$

Length of cross belt drive

$$L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}$$

$$L = \pi/_2(d_1+d_2) + 2x + \frac{(d_1+d_2)^2}{4x}$$

Power transmitted by a Belt

Let T_1 and T_2 - Tensions in the tight and slack side respectively in Newtons

r_2 and r_2 - Radii of driver and follower in 'm'

V - velocity of the belt in m/s

The effective turning force at the circumference of the follower.

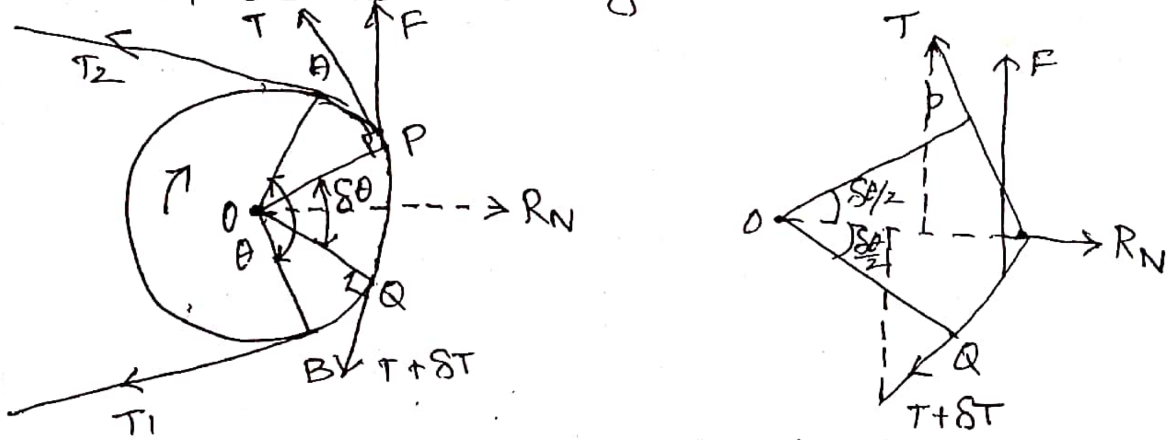
$$\text{workdone/sec} = (T_1 - T_2)\,/\text{sec}$$

$$\text{Power} = (T_1 - T_2) \times V \quad \text{Nm/s}$$

$$\boxed{P = (T_1 - T_2) \times V} \ \text{W.}$$

Ratio of driving tensions for flat belt drive

Consider a driven pulley rotating in the clockwise direction as shown in fig.



Let T_1 - Tension in the belt on tight side

T_2 - Tension in the belt on slack side

θ - Angle of contact in radians (Angle subtended by the arc AB)

Now consider ~~the~~ a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley. The belt PQ is in equilibrium under the following forces

1. Tension T in the belt at P
2. Tension $(T+\delta T)$ in the belt at Q
3. Normal reaction R_N
4. Frictional force $F = \mu R_N$, where μ - co-effi friction bet belt and pulley

Resolving all the forces horizontally

$$R_N = T\sin\delta\theta/2 + (T+\delta T)\sin\frac{\delta\theta}{2} \longrightarrow (1)$$

The angle $\delta\theta/2$ is very small $\sin\delta\theta/2 = \delta\theta/2$ in (1)

$$R_N = T\left(\frac{\delta\theta}{2}\right) + (T+\delta T) \times \frac{\delta\theta}{2}$$

$$= \frac{T\cdot\delta\theta}{2} + \frac{T\delta\theta}{2} + \frac{\delta T\cdot\delta\theta}{2}$$

$$R_N = \frac{2T\cdot\delta\theta}{2} \Rightarrow R_N = T\delta\theta$$

[illegible] resolving the forces vertically,

$$F = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$\frac{\delta\theta}{2}$ is very small, put $\cos\frac{\delta\theta}{2} = 1$ in above equ

$$F = \mu R_N = (T+\delta T) - T$$

$$\mu R_N = \delta T$$

$$R_N = T\cdot\delta\theta$$

$$\mu\, T\cdot\delta\theta = \delta T \Rightarrow T\cdot\delta\theta = \frac{\delta T}{\mu}$$

$$\cancel{T = \frac{\delta T}{\delta\theta\cdot\mu}} \quad \mu\,\delta\theta = \frac{\delta T}{T}$$

Integrating on both sides

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu\int\delta\theta$$

$$\log_e\left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\boxed{T_1/T_2 = e^{\mu\theta}}$$

Angle of contact or lap

$\theta = (180° - 2\alpha)\,\pi/180$ rad for open belt drive $\sin\alpha = \frac{r_1 - r_2}{x}$

$\theta = (180° + 2\alpha)\,\pi/180$ rad for cross belt drive $\sin\alpha = \frac{r_1 + r_2}{x}$

Centrifugal tension

$$T_C = mv^2$$

m – mass of belt/unit length

v – Linear velocity of belt

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Maximum tension in the belt

T = maximum stress × cross sectional area of the belt

$$\boxed{T = \sigma b t}$$

σ - maximum safe stress in N/mm^2
b - width of the belt in mm
t - Thickness of the belt in mm

Condition for the transmission of maximum power

WKT power transmitted by the belt

$$P = (T_1 - T_2) \times v \rightarrow (1)$$

T_1 - Tension in the tight side of the belt
T_2 - Tension in the slack side of the belt
V - Velocity of the belt in m/s

But WKT $\frac{T_1}{T_2} = e^{\mu\theta}$

$$\frac{T_1}{e^{\mu\theta}} = T_2 \rightarrow (2)$$

Put (2) in (1)

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) \times V = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) \times V$$

$$P = T_1 V C \quad \because C = \left(1 - \frac{1}{e^{\mu\theta}}\right) \rightarrow (3)$$

WKT $T_1 = T - T_c$

T - maximum tension
T_c - centrifugal tension

$$P = T_1 v c$$

Put $T_1 = T - T_c$

$$P = (T - T_c) v c$$

$$= (T - mv^2) \times v \times c$$

$$= Tvc - mv^3 c$$

$$P = c(Tv - mv^3)$$

For maximum power, differentiate the above equation w.r. to v and equate to zero,

$$\frac{dP}{dv} = 0$$

$$\frac{d}{dv} c(Tv - mv^3) = 0$$

$$T - 3v^2 = 0$$

$$T - 3T_c = 0 \qquad \because T_c = mv^2$$

$$\boxed{T = 3T_c}$$

when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

<u>Ratio of Driving tensions for V-belt</u>

$$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}\beta}$$

2β - Angle of groove.

11.8, 11.9, 11.10, 11.11, 11.12, 11.13, 11.19

1. A rope drive is required to transmit 230 kW ... pulley of 1m diameter running at 450 rpm. The safe pu... each rope is 800N and the mass of the rope is 0.4 kg per metre length. The angle of lap and the groove is 160° and 45° respectively. If $\mu = 0.3$ find the number of ropes required.

Given:-

$P = 230\,kW = 230 \times 10^3\,W$

$d = 1\,m$ $\quad 2\beta = 45^\circ$

$N = 450\,rpm$ $\quad \beta = 22.5^\circ$

$T = 800\,N$

$m = 0.4\,kg/m$

$\theta = 160^\circ = 2.792\,rad$

$\mu = 0.3$

To find:-

1. No of ropes ~~of~~ required

Solution:-

$$\text{Number of ropes required} = \frac{\text{Total power}}{\text{Power transmitted by one rope}}$$

$$P = (T_1 - T_2)\,v$$

$\cancel{T_1 = T_0 - T_c}$

$$T_1 = T - T_c$$

$$v = \pi d N / 60 = \frac{\pi \times 1 \times 450}{60} = 23.56\,m/s$$

$$T_c = mv^2$$

$$T_c = \underset{kg}{0.46} \times (\underset{m/sec^2}{23.56})^2$$

$$= 255.33\,N$$

$$T_1 = 800 - 255.33 = 544.67\,N$$

$$\frac{T_1}{T_2} = e^{\mu\theta\,\mathrm{cosec}\,\beta}$$

$$\frac{544.67}{T_2} = e^{0.3 \times 2.792 \times \mathrm{cosec}\,22.5}$$

$$\frac{544.67}{T_2} = 8.924$$

$$T_2 = 61.03\,N$$

$P = (T_1 - T_2) \times V$

$= (544.67 - 61.03) \times 23.56$

$P = 11.394$ kw

Number of ropes $= \frac{230}{11.394} = 20.18$

$\therefore n = 20$ ropes

(2). A compressor requires 90 kw to operate at 250 rpm. The drive is by V belts from an electric motor running at 750 rpm The dia of the pulley on the compressor shaft must not be greater than 1 metre while the center distance btn the pulleys is limitted to 1.75 m. The belt speed should not exceed 1600 m/m Determine the number of V belts required to transmit the power if each belt has a cross-sectional area of 375 mm². density 1000 kg/m³ and an allowable tensile stress of 2.5 MPa. The groove angle of the pulley is 35°, The μ btn the belt and pulley is 0.25. Also calculate the length required for each belt.

1 min = 60 sec

Given:-

$P = 90$ kw $\quad d_1 = 1$ m

$x = 1.75$ m

$N_1 = 250$ rpm $\quad V = 1600$ m/min

$N_2 = 750$ rpm $\quad = 26.66$ m/sec

$a = 375$ mm² $\quad T = 2.5$ MPa $= 2.5 \times 10^6$ N/m²

$= 375 \times 10^{-6}$ m² $\quad 2\beta = 35° \Rightarrow \beta = 17.5°$

$\rho = 1000$ kg/m³

$\mu = 0.25$

To find:- n, L

Solution:-

$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow \frac{750}{250} = \frac{1}{d_2}$

$3 = 1/d_2 \Rightarrow d_2 = 0.33$ m

$T_c = mV^2$

$m =$ Area $\times$ length $\times$ density $= 375 \times 10^{-6} \times 1 \times 1000$

$m = 0.375$ kg/m

$$T_c = mv^2$$
$$= 0.375 \times 26.66$$
$$T_c = 9.997 N$$
$$T = T_1 + T_c$$
$$T = 2.1 \times 10^6 \, N/m^2 \times 375 \times 10^{-6}$$
$$T = 787.5 N$$
$$T_1 = T - T_c$$
$$= 787.5 - 9.99$$
$$T_1 = 777.51$$
$$\theta = (180 - 2\alpha) \pi/180$$
$$\alpha = \frac{r_1 - r_2}{x} = 0.1914$$
$$\theta = 3.134 \, ra$$
$$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}\beta}$$
$$\frac{T_1}{T_2} = 13.53$$
$$\frac{777.51}{T_2} = 13.53$$
$$T_2 = 57.42 N$$
$$P = (T_1 - T_2) \times v$$
$$= 19197.5 \, w$$
$$n = 4.68 \Rightarrow 5 \text{ no's}$$
$$L = \pi(r_1 + r_2) + 2x + \frac{r_1 - r_2}{x}$$
$$L = 5.78 m$$

③. An open belt drive connects two pulleys 120cm and 50 cm dia on parallel shafts 4m apart. The maximum tension in the belt is 1855 N. The μ is 0.3. The driver pulley of dia 120 cm runs at 200 rpm calculate i). power transmitted ii). torque on each of the two shafts.

$$P = (T_1 - T_2) v$$
$$v = \pi d_1 N_1 / 60$$
$$T_L = (T_1 - T_2) r_1$$
$$T_S = (T_1 - T_2) r_2$$

[Deter]mine the width of a 9.75mm thick leather [b]elt required to transmit 15KW from a motor running at 900 rpm. The dia of driving pulley is 300 mm. The driven pulley runs at 300 rpm and the distance btn the centers of two pulley is 3m. The density of the leather can be taken as 1000 kg/m³. Take $\mu = 0.3$ and maximum allowable stress in the leather = 2.5 mPa and the drive is open type. (9)

m = Area x length x density

$= (b \times t) \times l \times \rho$

$T = \sigma b t$

$\frac{N_2}{N_1} = \frac{d_1}{d_2}$

$T = T_1 + T_c$

Given:-

$t = 9.75mm$ $N_2 = 300 rpm$

$P = 15KW$ $x = 3m$

$N_1 = 900 rpm$ $\rho = 1000 kg/m^3$

$d_1 = 300 mm$ $\mu = 0.3$

$\sigma = 2.5 \times 10^6 N/m^2$

$\frac{T_1}{T_2} = e^{\mu \theta}$

(5). An open belt running over a two pulleys 1.5m and 1m dia connects two parallel shaft 4.80m apart. The initial tension in the belt when stationary is 3000N. If the smaller pulley rotating at 600 rpm and μ btn belt and pulley is 0.3. Determine power transmitted taking T_c into account. The mass of the belt is given as 0.673 kg/length.

Initial tension $T_0 = \frac{T_1 + T_2 + 2T_c}{2}$

(6). Two pulleys one 450 mm dia and the other 200 mm dia are on parallel shafts 2.1m apart and are connected by a cross belt. The larger pulley rotates at 225 rpm. The maximum permissible tension in the belt is 1KN and μ btn the belt and pulley is 0.25. Find, the length of the belt required and the power that can be transmitted. $T_1 = 1KN$

Q. Determine the width of a 9.75mm [illegible] required to transmit 15kW from a motor running at 900 rpm. The [illegible] of the driving pulley of motor is 300mm [illegible] driven pulley runs at 300 rpm and the [illegible] centres of two pulleys is 3m. The [illegible] is 1000 kg/m³. The max allowable stress in the [illegible] is 2.5 MPa. The μ between the contact and [illegible] is 0.3. Assume open belt drive and neglect [illegible] the belt.

GD:-

$t = 9.75mm$ $\quad$ $P = 15\times10^3 W$, $N_1 = 900 rpm$, $d_1 = 0.3 m$, $N_2 = 300$

$= 9.75\times10^{-3}$ $\quad$ $x = 3m$, $\rho = 1000 kg/m^3$, $\sigma = 2.5\times10^6 N/m^2$

$\mu = 0.3$

To find:-

width of belt

solution.

WKT, max tension in the belt $T = \sigma b t$

$$T = T_1 + T_c$$

$$P = (T_1 - T_2) \times V$$

$$V = \frac{\pi d_1 N_1}{60} = \frac{\pi\times0.3\times900}{60} = 14.14 m/s$$

$$\frac{T_1}{T_2} = e^{\mu\theta}, \quad \theta = 180 - 2\alpha \times\pi/180 \text{ for open belt drive}$$

$\theta = 2.942$

$$\sin\alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x}$$

m = area x length x density

$= btl\rho$

$T_c = mv^2$

$=$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow d_2 = \frac{N_1\times d_1}{N_2} = \frac{900\times0.3}{300} = 0.9$$

$$\sin\alpha = \frac{d_2 - d_1}{2x} = \frac{0.9 - 0.3}{2\times3} = 0.1$$

$$\alpha = \sin^{-1}0.1 = 5.74^\circ$$

$$\frac{T_1}{T_2} = e^{\mu\theta} = 2.42 \Rightarrow T_1 = 2.42T_2$$

$$P = (T_1 - T_2)\times V \Rightarrow 15\times10^3 = (T_1 - T_2)14.14 \Rightarrow T_1 - T_2 = 1060N$$

$$T_1 = 1806N$$

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Brakes

Definition:-

A brake is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy.

A dynamometer is a brake incorporating a device to measure the frictional resistance applied.

Types of Brake (mechanical)

1). Block or shoe Brake
2). Band Brake
3). Band and Block brake
4). Internal expanding shoe brake

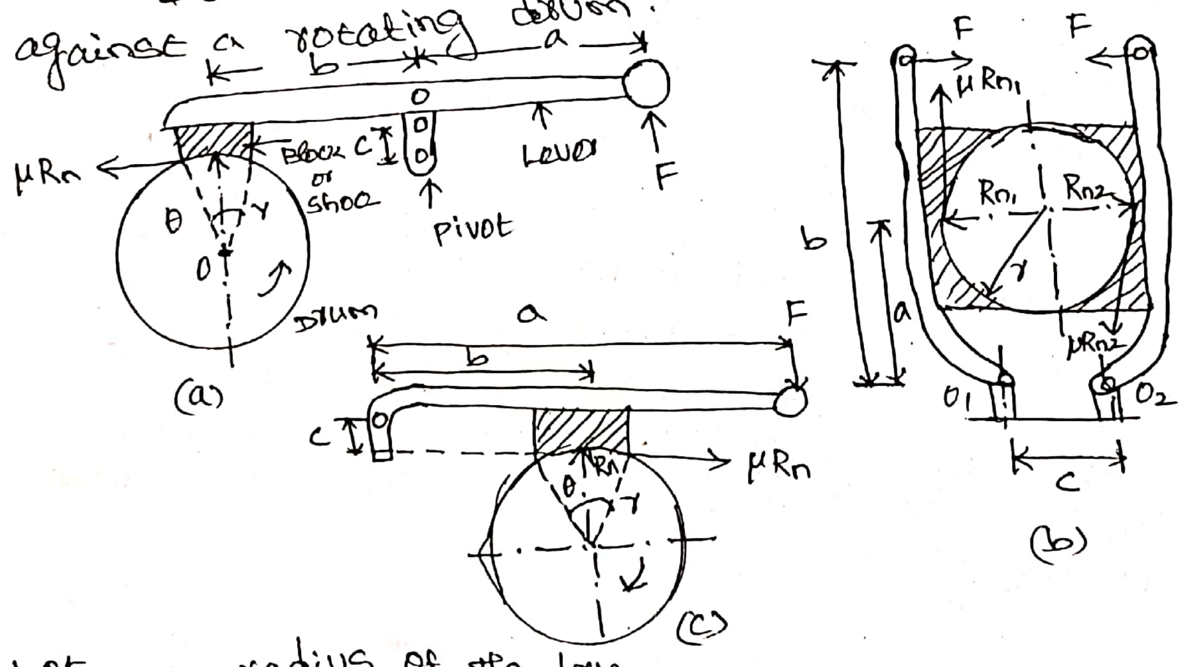
General

1. Hydraulic brakes
2. Electric brakes
3. Mechanical brakes
   a). Radial brake
   - block or shoe brake
   - band brake

   b). Axial brake
   - disc & cone

Block or shoe Brake

It consists of a block or shoe which is pressed against a rotating drum.



Let r - radius of the drum
μ - Co-efficient of friction
R_n - normal reaction on the block
F - force applied at the lever end
F_f - frictional force $= \mu R_n$
F_r - radial force applied on the drum

Assume, Rn and F_f act at the mid point of the ... braking torque on the drum = frictional force × radius

$T_B = \mu R_n \times r$

Taking moment about the pivot 'O'

Consider fig (a).

$F \times a - R_n \times b + \mu R_n \times c = 0$

$Fa = R_n b - \mu R_n c$

$Fa = R_n (b - \mu c)$

$R_n = \dfrac{Fa}{b - \mu c}$

$F = R_n \left[\dfrac{(b - \mu c)}{a}\right]$ For clockwise rotation

when $b = \mu c$, $F = 0$ → self locking brake

$F = R_n \left[\dfrac{b + \mu c}{a}\right]$ For ccw rotation

From fig (c)

$\mu' = \mu \left(\dfrac{4 \sin\theta/2}{\theta + \sin\theta}\right)$

1. A bicycle and rider travelling at 12 km/h on a level road, have a mass of 105 kg. A brake is applied to the rear wheel which is 800 mm in diameter. The pressure on the brake is 80 N and the μ is 0.06. Find the distance covered by the bicycle and number of turns of its wheel before coming to rest.

Given:-

Velocity = 12 km/hr = $\dfrac{12 \times 1000}{3600}$ = 3.33 m/s

Dia d = 800 mm = 0.8 m

$\mu = 0.06$

radial force → $F_r = 80N$

To find:-

1). Distance covered

2). Number of turns

Let S = distance covered by the bicycle before it comes to rest

Work done against friction = KE of the bicycle and rider

Force x distance = $\frac{1}{2}mv^2$

$\mu R_n \times S = \frac{1}{2}mv^2$

$0.06 \times 80 \times S = \frac{1}{2} \times 105 \times 3.33^2$

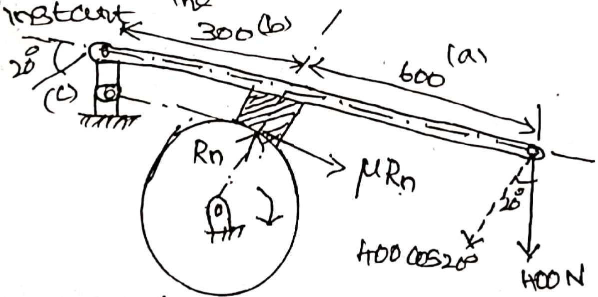
$$\boxed{S = 121.45\,m}$$

$S = \pi d n$ = circumference x number of turns

$121.45 = \pi \times 0.8 \times n$

$\boxed{n = 48.32}$ revolutions

(2). A brake drum of 440 mm in dia is used in a braking system as shown in fig. The brake lever is inclined at an angle of 20° with the horizontal. A vertical force of 400 N magnitude is applied at the lever end. The μ is 0.35. The brake drum has a mass of 160 kg and its rotates at 1500 rpm. Determine the i). braking torque ii). number of revolutions made by the drum and the time taken before coming to rest from the instant the brake is applied.



Given:-

$d = 440\,mm$ $\quad m = 160\,kg$

$\mu = 0.35$ $\quad F = 400\cos 20°$

$N = 1500\,rpm$

To find:- T_B, n, t

Solution:-

Taking moment about fulcrum

$400\cos 20° \times 900 + \mu R_n \times c - R_n \times 300 = 0$

$338289.34 + \mu R_n \times 0 - 300 R_n = 0$

$R_n = 1127.63\,N$

Braking torque $T_B = \mu R_n \times r$

$= 0.35 \times 1127.63 \times 0.220$

$T_B = 86.82\ Nm$

workdone against friction = K.E of the brake drum

$T_B \times \omega = \frac{1}{2} m v^2$

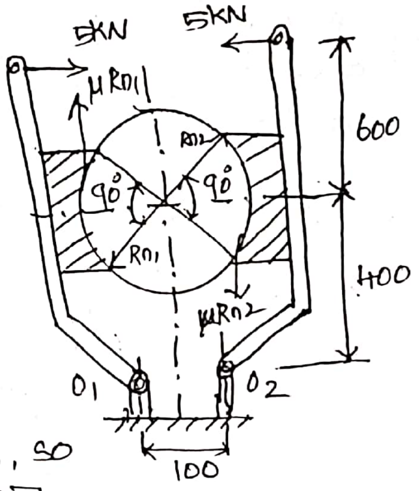
$86.82 \times \omega = \frac{1}{2} \times 160 \times \left(\frac{0.44 \times \pi \times 1500}{60}\right)^2$

$86.82 \times 2\pi n = 95537.77$

$n = 175$ revolutions

Time taken $t = \frac{n}{N} = \frac{175}{1500/60} = 7$ seconds.

(3). A spring operated pivoted shoe brake shown in fig. The wheel dia is 500mm. The angle of contact is 90° and μ is 0.3. The force applied by the spring on each arm is 5KN. Determine the brake torque on the wheel.



Given:-

$d = 500mm$

$\theta = 90°$

$\mu = 0.3$

$F = 5KN$

To find:- T_B

Solution:-

Braking torque $T_B = \mu'(R_{n1} + R_{n2}) \times r$, As the θ more than 40°, so

$\mu' = \mu \left[\frac{4\sin\theta/2}{\theta + \sin\theta}\right] = 0.3 \left(\frac{4 \sin 45°}{\pi/2 + \sin 90°}\right) = 0.329$

For the left hand side block, taking moment about O_1

$5000 \times 1 - R_{n1} \times 0.4 + \mu' R_{n1}(0.25 - 0.05) = 0$

$5000 - 0.4 R_{n1} + 0.0658 R_{n1} = 0$

$R_{n1} = 14961.1N$

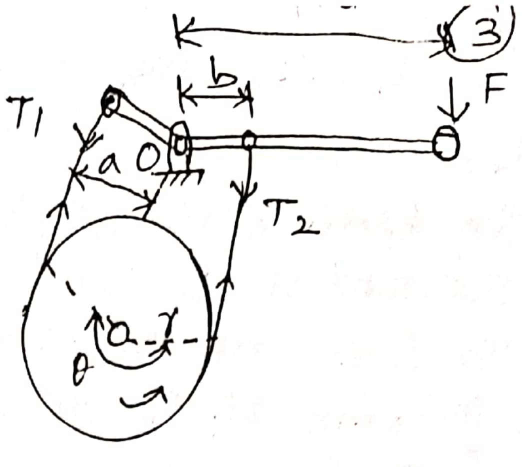
For the right-hand side block, taking moment about O_2

$5000 \times 1 - R_{n2} \times 0.4 - \mu' R_{n1}(0.25 - 0.05) = 0$

$R_{n2} = 10734\ N$

$\therefore T_B = \mu'(R_{n1} + R_{n2}) \times r = 2113.41\ Nm$

...nd Brake



It consists of rope, belt or flexible steel band (lined with friction material) which is pressed against the external surface of a cylindrical drum when the brake is applied.

Brake torque on the drum $= (T_1 - T_2) \times r$

i). when $a > b$ → F is applied in downward direction

ii). when $a < b$ → F is applied in upward direction

- $a > b$, $F \downarrow$

Taking moments about the pivot

ccw

$$Fl - T_1 a + T_2 b = 0$$

$$F = \frac{T_1 a - T_2 b}{l}$$

cw

$$Fl - T_2 a + T_1 b = 0$$

$$F = \frac{T_2 a - T_1 b}{l}$$

- $a < b$, $F \uparrow$

ccw

$$Fl + T_1 a - T_2 b = 0$$

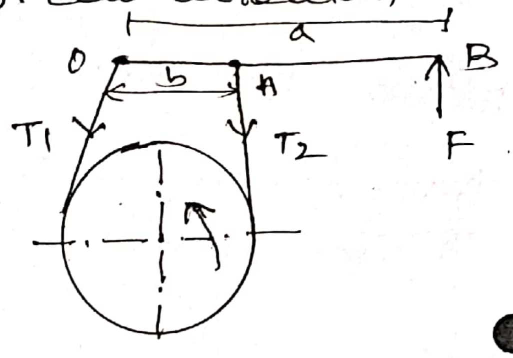
$$F = \frac{T_2 b - T_1 a}{l}$$

cw

$$Fl + T_2 a - T_1 b = 0$$

$$F = \frac{T_1 b - T_2 a}{l}$$

1. A band brake acts on the 3/4 of circumference drum of 450 mm diameter which is keyed to the sh[aft]. The band brakes provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force is applied at 500 mm from the fulcrum and μ is 0.25. Find the operating force when the drum rotates in a). ccw direction b). cw direction.



Given:-

Angle of contact $\theta = 3/4$ of circumference

$T_B = 225\,Nm$

$b = 100\,mm$

$a = 500\,mm$

$d = 450\,mm$

$\mu = 0.25$

To find:-

operating force when the drum rotates

i). ccw ii). cw

Solution:-

Let F = operating force

$$\theta = \left(\tfrac{3}{4} \times 360^\circ\right) \times \pi/180 = 4.712\,rad$$

i). when the drum rotates in ccw direction

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.2 \times 4.712} = 3.247$$

$$T_1 = 3.247\,T_2$$

$$T_B = (T_1 - T_2) \times r$$

$$225 = (3.247\,T_2 - T_2) \times 0.225$$

$$T_2 = 445\,N$$

$$T_1 = 1445\,N$$

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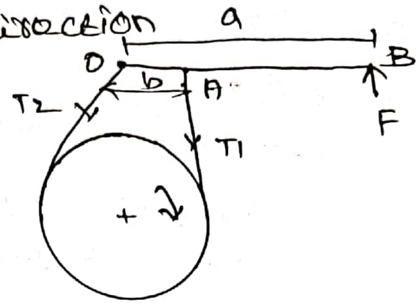
Taking moment about O

$Fa = T_2 b$

$F \times 0.5 = 445 \times 0.1$

$\boxed{F = 89\,N}$

ii). When drum rotates in ccw direction



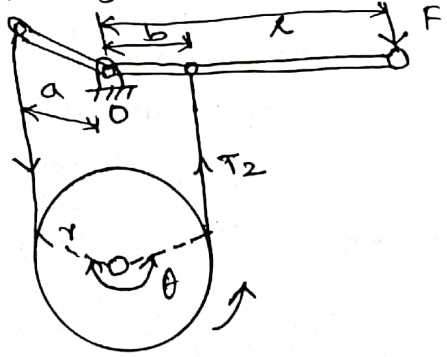
Taking moment about 'O'

$Fa = T_1 b$

$F \times 0.5 = 1445 \times 0.1$

$\boxed{F = 289\,N}$

(2). A differential band brake has a drum with a diameter of 800mm. The two ends of the band are fixed to the pins on the opposite sides of the fulcrum of the lever at distances of 40 mm and 200 mm from the fulcrum. The angle of contact is 270° and the μ is 0.2. Determine the brake torque when a force of 600 N is applied to the lever at a distance of 800 mm from the fulcrum.



Given:-

$d = 800\,mm$

$a = 40\,mm$, $b = 200\,mm$

$l = 800\,mm$

$\theta = 270^\circ$, $\mu = 0.2$

$F = 600\,N$

To find:-

Braking Torque

Solution:-

Assume $a > b$, F must act downwards to apply brake

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.2 \times 270 \times \pi/180} = 2.566$$

$$T_1 = 2.566\, T_2$$

$a = 0.2$
$b = 0.04$

ccw direction of the drum

Taking moments about the fulcrum,

$$Fl + T_2 b - T_1 a = 0$$

$$600 \times 0.8 + T_2 \times 40 - 2.566 T_2 \times 0.2 = 0$$

$$480 + \overset{0.04T_2}{\cancel{40T_2}} - 0.5132 T_2 = 0$$

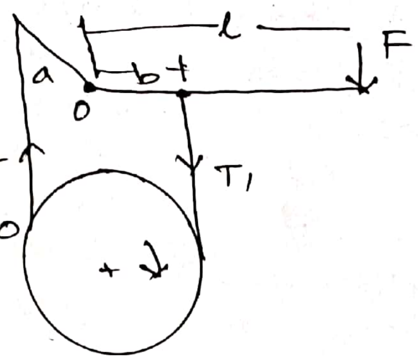
$$T_2 = 1014.3\,N$$

$$T_1 = 2602.8\,N$$

$$T_B = (T_1 - T_2) \times r$$

$$\boxed{T_B = 635.4\ Nm}$$

Cw direction of the drum



Taking moment about 'O'

$$Fl + T_1 b - T_2 a = 0$$

$$600 \times 0.8 + 0.04 \times 2.566 T_2 - T_2 \times 0.2 = 0$$

$$480 + 0.1026 T_2 - 0.2 T_2 = 0$$

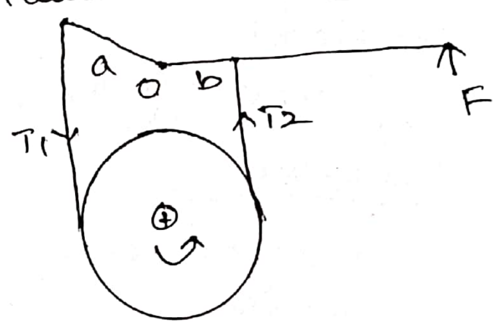
$$T_2 = 4930.1\,N$$

$$T_1 = 12650.7\,N$$

$$T_B = (T_1 - T_2) \times r = 3088.24\ Nm$$

$$\boxed{T_B = 3088.24\ Nm}$$

Assume $a < b$, F must be acc upwards to apply brake



ccw

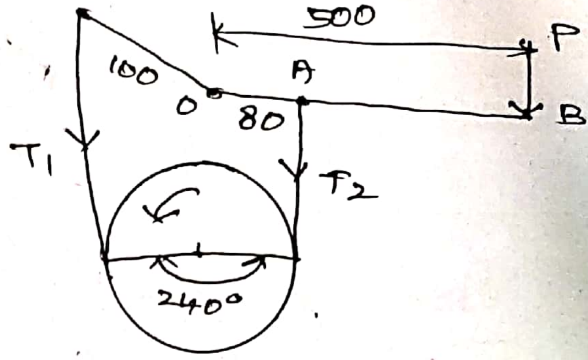
Taking moment about 'O'

$$Fl + T_1 a - T_2 b = 0$$

cw

$$Fl + T_2 a - T_1 b = 0$$

... differential band brake is operated by a lever as shown in fig. The brake drum has a dia of 500 mm and the maximum torque on the drum is 1 kNm. If the μ b/n brake lining and drum is 0.3, find the operating force. (2) (5).



Given:-

$d = 500$ mm

$l = 500$ mm

$a = 100$ mm

$b = 80$ mm

$T_B = 1$ kN-m

$\mu = 0.3$

$\theta = 240°$

TO find:-

operating force

solution:-

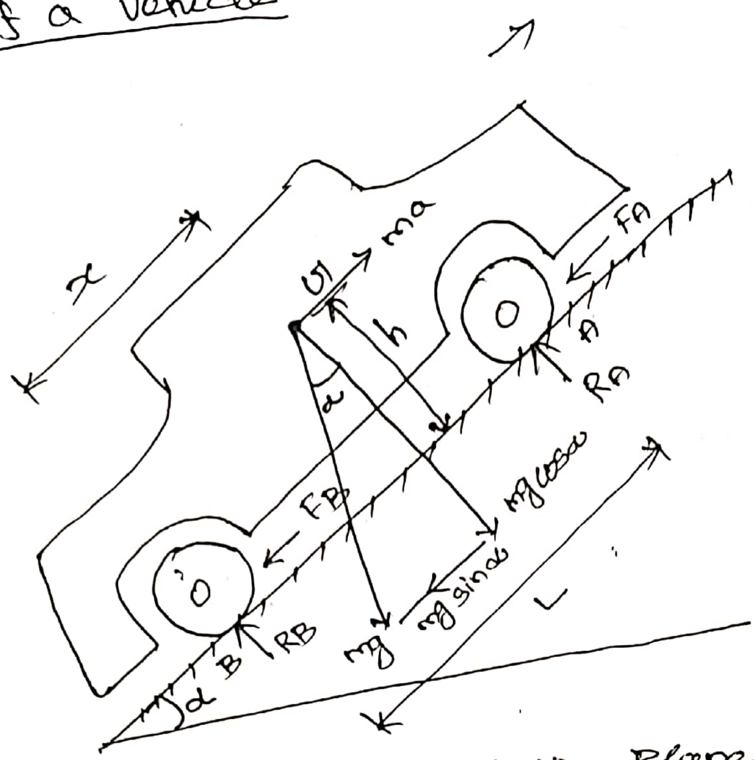
$$\frac{T_1}{T_2} = e^{\mu\theta}, \quad T_B = (T_1 - T_2) \times r$$

Taking moment about 'O'

$$P \times 0.5 + T_2 80 - T_1 \times 100 = 0$$

$$P \times 0.5 = T_1 \times 100 - T_2 \times 80$$

Braking of a vehicle



Let α - Angle of inclination of the plane to the horizontal

m - mass of the vehicle

h - Height of C.G of the vehicle above the road surface

x - ⊥ᶫᵗ distance of C.G from the rear axle

L - wheel base

RA - Total normal reaction btn the ground and front wheel

RB - " " " " and rear wheel

FA - Total braking force acting at the front wheels due to the application of brakes

FB - Total braking force acting at the rear wheels due to the application of brakes.

μ - btn tyres and road surface

a - Retardation of the vehicle.

1). When the brakes are applied to the front wheels only

$$a = \frac{\mu \cdot g \cos\alpha \, x}{L - \mu h} + g \sin\alpha$$

$$R_A = \frac{mg \cos\alpha \, x}{L - \mu h}$$

$$R_B = mg\cos\alpha \left[\frac{L - \mu h - x}{L - \mu h}\right]$$

$$a = g(\mu \cdot \cos\alpha + \sin\alpha)$$

$$R_A = \frac{m \cdot g \cos\alpha (\mu h + x)}{L}$$

$$R_B = m g \cos\alpha \left[\frac{L - \mu h - x}{L}\right]$$

(1). A car moving on a level road at a speed 50 km/hr has a wheel base 2.8 m, distance of C.G from ground level 500 mm and the distance of C.G from rear wheels 1.1 m. Find the distance travelled by the car before coming to rest when the brakes are applied
a). to the rear wheels b). to the front wheels
c). to all the four wheels and μ is 0.50

Given:-

$\alpha = 0°$

$u = 60$ km/hr $= \frac{60 \times 1000}{3600}$ m/s $= 16.67$ m/s

$S = \frac{u^2}{2a}$

$L = 2.8$ m

$h = 500$ mm

$x = 1.1$ m

$\mu = 0.5$

To find:-

Distance travelled by the car before coming to rest when the brakes are applied

1). to the rear wheels
2). to the front wheels
3). to all the four whe

solution

Let S – Distance travelled by the car before coming to rest

a – Retardation of the car.

1). when the brakes are applied to the rear wheels only

$$\cancel{a} = \cancel{\ } u^2 + 2as =$$

$$u^2 = 2as$$

$$S = u^2/2a$$

$$a = \frac{\mu \cdot g \cos\alpha (L - x)}{L + \mu h} + g \sin\alpha$$

$$a = \frac{0.5 \times \overset{9.81}{\cancel{0.981}} \cos 0^\circ (2.8 - 1.1)}{2.8 + (0.5 \times 0.5)} + \overset{9.81}{\cancel{0.981}} \sin 0$$

$$a = 2.73 \, m/s^2$$

$$\therefore S = \frac{16.67^2}{2 \times 2.73} = 50.89 \, m$$

2). when the brakes are applied to the front wheels only

$$a = \frac{\mu \cdot g \cos\alpha \times x}{L - \mu h} + g \sin\alpha$$

$$= \frac{0.5 \times 9.81 \times \cos 0^\circ \times 1.1}{2.8 - (0.5 \times 0.5)} + 9.81 \times \sin 0$$

$$a = 2.12 \, m/s^2$$

$$S = u^2/2a = \frac{16.67^2}{2 \times 2.12} = 65.53 \, m$$

3). when the brakes are applied on all four wheels

$$a = g \, \mu (\cos\alpha + \sin\alpha)$$

$$= 9.81 \times [0.5 \cos 0^\circ + \sin 0^\circ)$$

$$= 4.905 \, m/s^2$$

$$S = u^2/2a = \frac{16.67^2}{2 \times 4.905} = 28.33 \, m$$

may/June 2007 15. b.(ii).

Given data:-

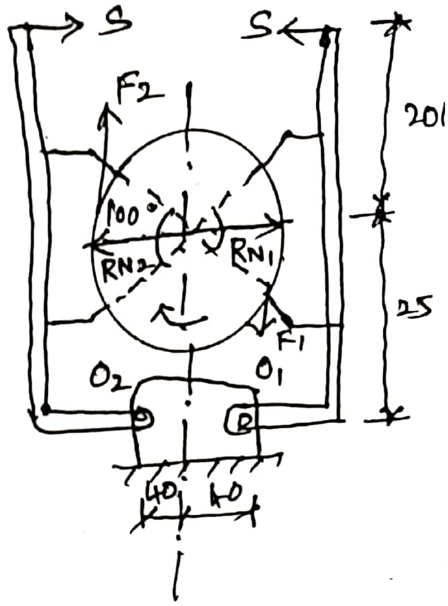
$\mu = 0.3$ &

$p = 1000\ kPa = 1\times10^5\ N/m^2$

$d = 200mm,\ r = 100\ mm$

$2\theta = 100^\circ \Rightarrow 2\theta = 100\times\pi/180 = 1.75\ rad$

$S = 1750N$



To find:-

1). Face width of the shoe (b)

2). braking torque (T_B)

Solution:-

$$T_B = (F_1+F_2)\times r$$

Equivalent $\mu' = \dfrac{4\mu\sin\theta}{2\theta+\sin2\theta}$ $[\because 2\theta > 40^\circ]$

$$p = \frac{RN_2}{Ab}$$

bearing area of shoe $Ab = b(2r\sin\theta)$

19

Friction clutch

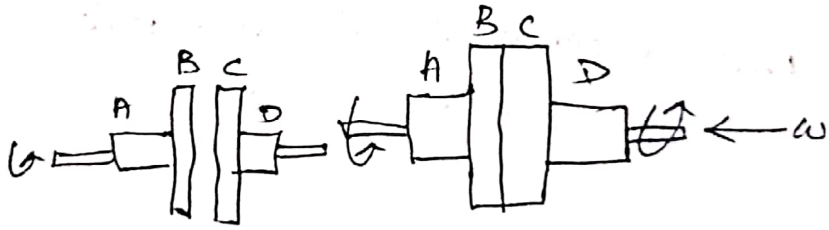
Definition:-

- A transmission device of an automobile which is used to engage and disengage the power from the engine to the rest of the system.

Functions:-

- supplies the power to the transmission system
- stops the vehicle
- change the gear and idling the engine
- gives gradual increment of speed to the wheels

* The ~~are~~ friction btw the two surfaces depends upon the area of the surfaces, pressure applied upon them and μ of the surface materials.



Types

1). Disc (or) plate clutches

a). single plate b). Multiplates

2). Cone clutch

3). Centrifugal clutch

2 - 29, 43, 53
3 - 11, 13, 43

Multiplate clutch

$$\boxed{T = n \mu W R}$$

where

T - Torque transmitted

n - Number of pair of contact surfaces.

$(n = n_1 + n_2 - 1)$

n_1 - number of discs on the driving shaft

n_2 - number of discs on the driven shaft.

Axial force to engage the clutch $\boxed{W = 2\pi C (r_1 - r_2)}$

$$\boxed{C = \frac{W}{2\pi (r_1 - r_2)}}$$

Average pressure on the friction surface

$$\boxed{P_{av} = \frac{W}{\pi (r_1^2 - r_2^2)}}$$

Number of friction surface required = $\frac{\text{Total torque required}}{\text{Torque required/surface}}$

Total number of plates = Number of pairs of contact surface + 1

Cone clutch

Torque transmitted by cone clutch

$$\boxed{T = \mu W R \operatorname{cosec} \alpha}$$

r - Distance of centre of gravity of the shoe from the centre of the spider

R - Inside radius of pulley rim.

$$\boxed{F_c - F_s = lbp}$$

Where, l - contact length of the shoes

b - width of the shoe

p - Intensity of pressure exerted ~~by~~ on the shoes.

1. May / June 2012 (15). a. (1).

Given:-

$d_1 = 300\,mm$

$d_2 = 200\,mm$

$P_{max} = 0.1\,N/mm^2$

$\mu = 0.3$

$N = 2500\,rpm$

To find:-

Power transmitted ~~in~~

1). uniform pressure

2). uniform wear

Solution:-

$$P = \frac{2\pi NT}{60}$$

$T = n\mu WR$, $n = 2$ (both sides effective) where $R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$ uniform pressure.

$R = \frac{r_1 + r_2}{2}$ uniform wear

$W = 2\pi C(r_1 - r_2)$

$C = P_{max} \cdot r_2$ (For uniform wear p is max at the inner radius).

$C = P_{max} \cdot r_1$ [For uniform pressure p is max at the outer radius.]

– x –

Nov/Dec 2011 (15). b. (i).

Given :-

$r_1 = 120\,mm$

$r_2 = 60\,mm$

$W = 1500\,N$

To find :-

For uniform wear

i). P_{max} (ii). P_{min} (iii). P_{av}

Solution :-

$P_{max} \cdot r_2 = C$

$P_{max} = C/r_2$

$C = \frac{W}{2\pi(r_1 - r_2)}$

$P_{min} \cdot r_1 = C$

$P_{min} = C/r_1$

$$P_{av} = \frac{W}{\pi\left[r_1^2 - r_2^2\right]}$$

— x —

(3) . May / June 2006 15. b. (ii).

Given:-

$D = 150mm$, $R = 75mm$ $\left[\because D/2 = R\right]$

$W = 20\ KN$

Cone angle $2\alpha = 120^\circ$, $\alpha = 60^\circ$

$\mu = 0.03$

$N = 200\ rpm$

To find:-

power lost in friction for uniform wear

Solution:-

$$P = \frac{2\pi NT}{60}$$

$$T = \mu W R \operatorname{cosec}\alpha$$

$$R = \frac{r_1 + r_2}{2} \quad (or) \quad R = \tfrac{1}{2}(r_1 + r_2)$$

$$\therefore T = \tfrac{1}{2}\mu W R \operatorname{cosec}\alpha$$

2-2

3-

④. May/June 2007 15.(a)

Given:-

uniform pressure

$d_2 = 300\,mm$, $r_2 = 150\,mm$

$W = 200\,KN$

$N = 75\,rpm$

$\mu = 0.05$

$p = 0.3\,N/mm^2$

$P = 16\,KW$

To find:-

uniform pressure condition.

i). d_1 ii). Number of collars

Solution:-

$$T = \mu W R \quad , \quad R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$\therefore T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

$$P = \frac{2\pi N T}{60} \Rightarrow T = \frac{P \times 60}{2\pi N}$$

$$\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} = \frac{(r_1 - r_2)\left[r_1^2 + r_2^2 + r_1 . r_2\right]}{(r_1 + r_2)(r_1 - r_2)} = \frac{r_1^2 + r_2^2 + r_1 . r_2}{r_1 + r_2}$$

$$p = \frac{W}{n\pi(r_1^2 - r_2^2)}$$

$$n = \frac{W}{p\pi(r_1^2 - r_2^2)}$$

5. May/June 2007. 15. a. (ii).

Given:-

$2\alpha = 30^\circ$, $\alpha = 15^\circ$

$p_n = 0.35\ N/mm^2$

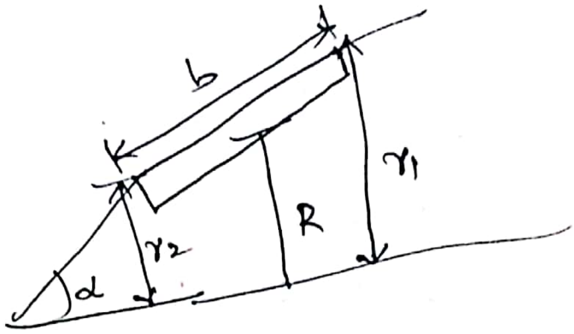
$b = R/3$

$P = 22.5\ kW$

$N = 2000\ rpm$

$\mu = 0.15$

Uniform wear



To find:-

r_1, r_2

$r_1 = 103.27\ mm$

$r_2 = 94.73\ mm$

<u>Solution:-</u>

$\cancel{T = 2\pi \mu p_n . R^2 . b}$

$P = 2\pi NT/60$

$\dfrac{r_1 - r_2}{b} = \sin\alpha$, $R = \dfrac{r_1 + r_2}{2}$

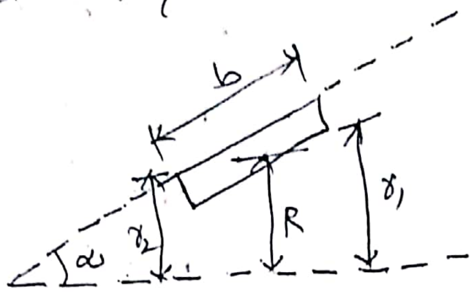
$p_{max} . r_2 = C$

$T = \mu W \operatorname{cosec}\alpha \left[\dfrac{r_1 + r_2}{2}\right]$

$W = 2\pi C (r_1 - r_2)$

Scanned by CamScanner

(1). A leather faced conical friction clutch has a cone angle of 30°. If the intensity of pressure between the contact surface is limited to 0.35 MPa and the breadth of the conical surface is ~~limited to~~ not to exceed one third of the mean radius, find the dimensions of contact surfaces to transmit 22.5 kW at 2000 rpm. Assume uniform rate of wear and take $\mu = 0.15$



Given:- Uniform wear

$2\alpha = 30°$ $\quad P_n = 0.35$ MPa

$= 0.35 \times 10^6$ N/m²

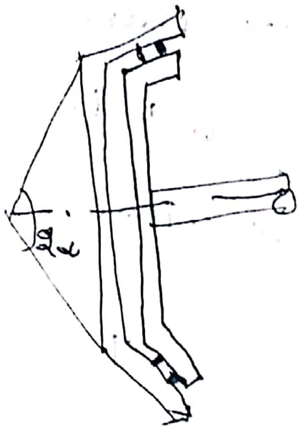
$\alpha = 15°$

$b = R/3$ $\quad P = 22.5$ kW

$= 22.5 \times 10^3$ W

$N = 2000$ rpm

$\mu = 0.15$



To find:-

1). b 2). r_1 3). r_2

Solution:-

WKT $\quad T = \mu W R \operatorname{cosec}\alpha$

$\frac{1}{\sin} = \operatorname{cosec}$

$\frac{1}{\cos} = \sec$

$1/\tan = \cot$

$W = 2\pi C (r_1 - r_2)$

$C = P_{max} \cdot r_2 = 0.35 \times 10^6 r_2$

$$\frac{r_1 - r_2}{b} = \sin\alpha$$

$$b = \frac{r_1 - r_2}{\sin\alpha}, \quad b = R/3 = \frac{1}{3}\,\frac{r_1 + r_2}{2} = \frac{r_1 + r_2}{6}$$

$$\frac{r_1 + r_2}{6} = \frac{r_1 - r_2}{\sin 15°} \Rightarrow \cancel{r_1 + r_2 =}$$

$$\frac{r_1 + r_2}{6} = \frac{r_1 - r_2}{0.258}$$

$$r_1 + r_2 = \frac{r_1 - r_2}{0.258} \times 6 \Rightarrow r_1 + r_2 = 23.25(r_1 - r_2)$$

$$r_1 + r_2 = 23.18 r_1 - 23.18 r_2$$

$$r_2 + 23.18 r_2 = 23.18 r_1 - r_1$$

$$24.18 r_2 = 22.18 r_1$$

$$r_2 = \frac{22.18 r_1}{24.18} = 0.917 r_1 \rightarrow (1)$$

$$W = 2\pi C (r_1 - r_2)$$

$$= 2\pi \times 0.35 \times 10^6 \, r_2 (r_1 - r_2)$$

$$W = 2.199 \times 10^6 \, r_2 (r_1 - r_2) \rightarrow (2)$$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{P \times 60}{2\pi N} = \frac{22.5 \times 10^3 \times 60}{2 \times \pi \times 2000} = 107.43 \text{ Nm}$$

$$\therefore T = \mu W R \operatorname{cosec} \alpha$$

$$107.43 = 0.15 \times \left[2.199 \times 10^6 \, r_2 (r_1 - r_2)\right] \cdot \frac{r_1 + r_2}{2} \cdot \operatorname{cosec} 15^\circ$$

$$= 0.15 \times 2.199 \times 10^6 \times 0.917 r_1 (r_1 - 0.917 r_1) \cdot \frac{r_1 + 0.917 r_1}{2} \times \operatorname{cosec} 15^\circ$$

$$= 0.15 \times 2.199 \times 10^6 \times 0.917 r_1 (0.083 r_1) \cdot \frac{1.917 r_1}{2} \cdot 3.86$$

$$= 0.15 \times 2.199 \times 10^6 \quad 92884.519 \, r_1^3$$

$$107.43 = \cancel{0.5 \times 0.15 \times} \; 92884.519 \, r_1^3$$

$$r_1^3 = \frac{107.43}{92884.51} = 1.156 \times 10^{-3}$$

$$r_1 = (1.156 \times 10^{-3})^{1/3}$$

$$r_1 = 0.107 \text{ m}$$

$$r_2 = 0.917 r_1 = 0.098 \text{ m}$$

$$b = \frac{R}{3} = \frac{1}{3}\left(\frac{r_1 + r_2}{2}\right) = \frac{0.107 + 0.098}{6} = 0.034 \text{ m}$$